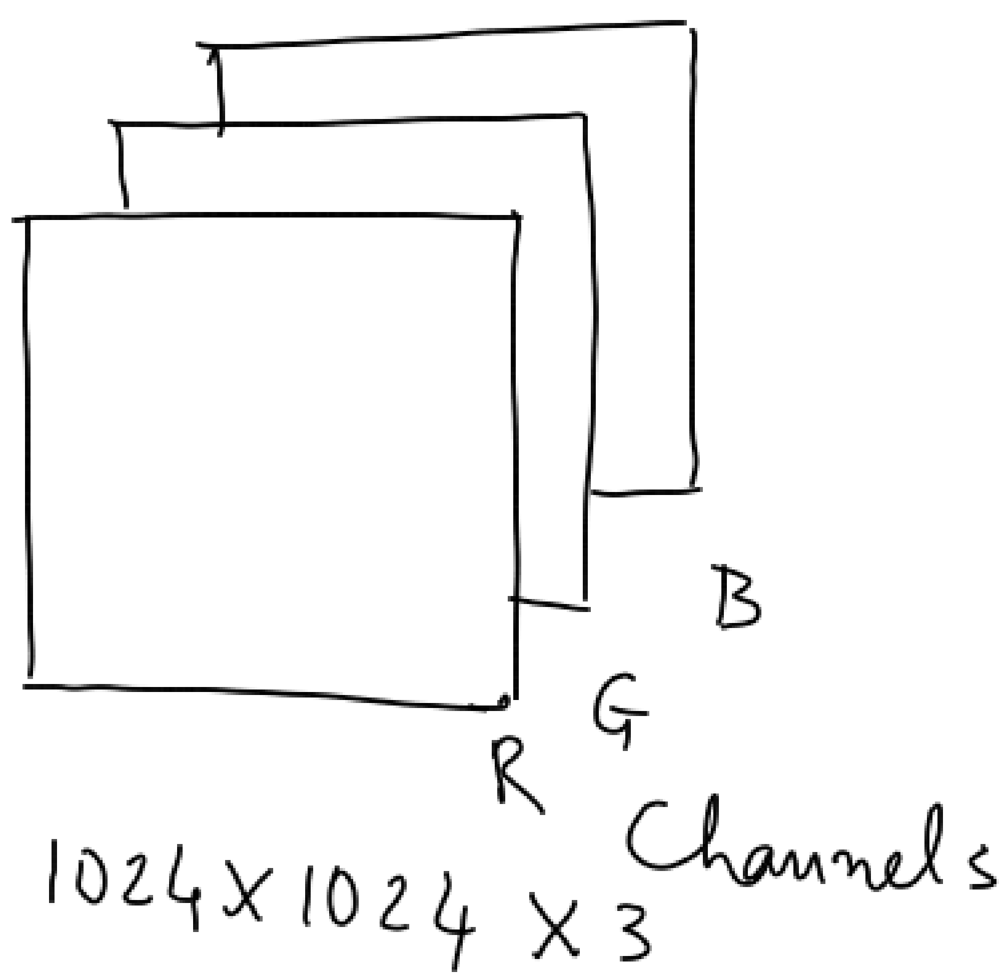
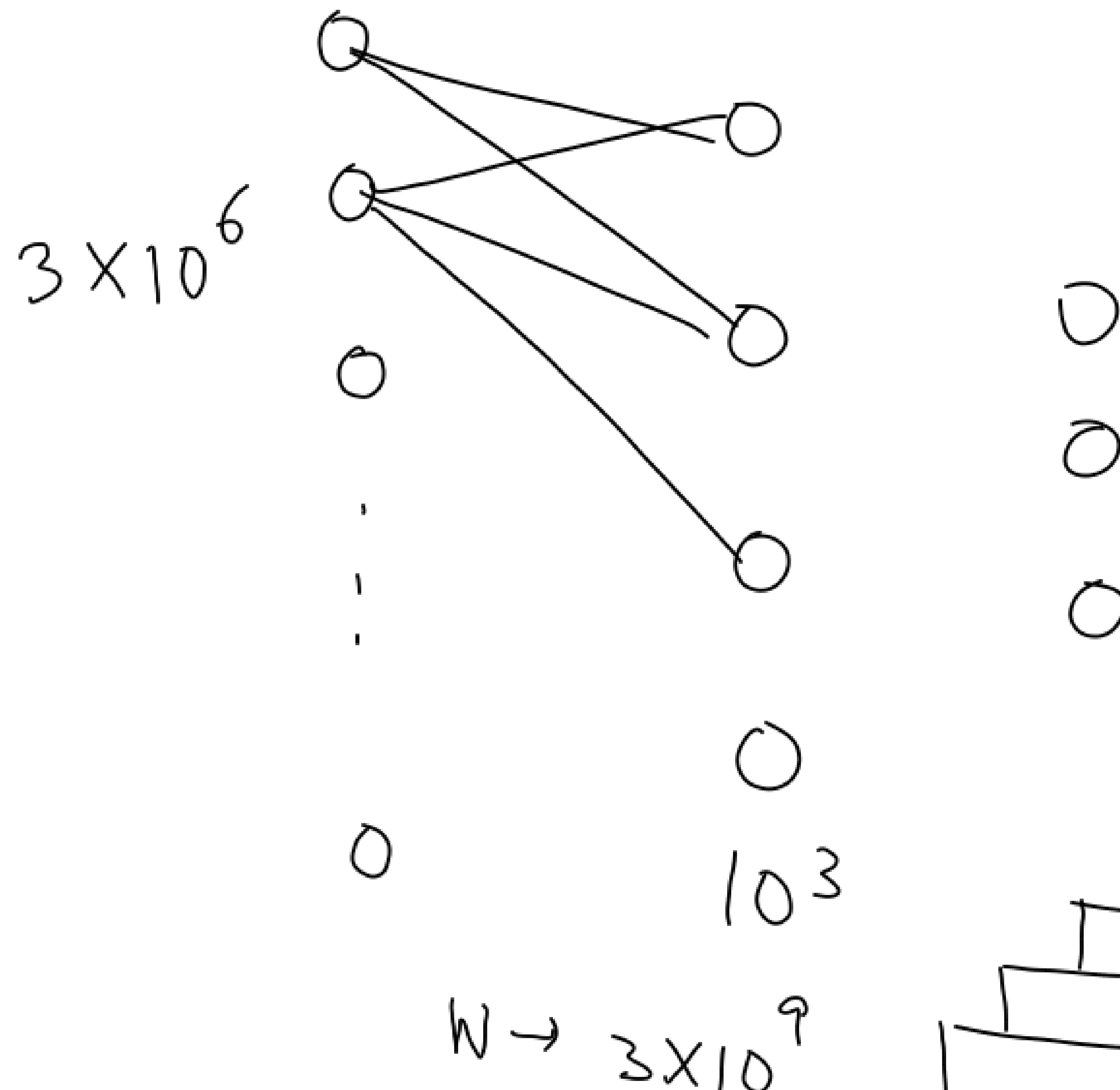
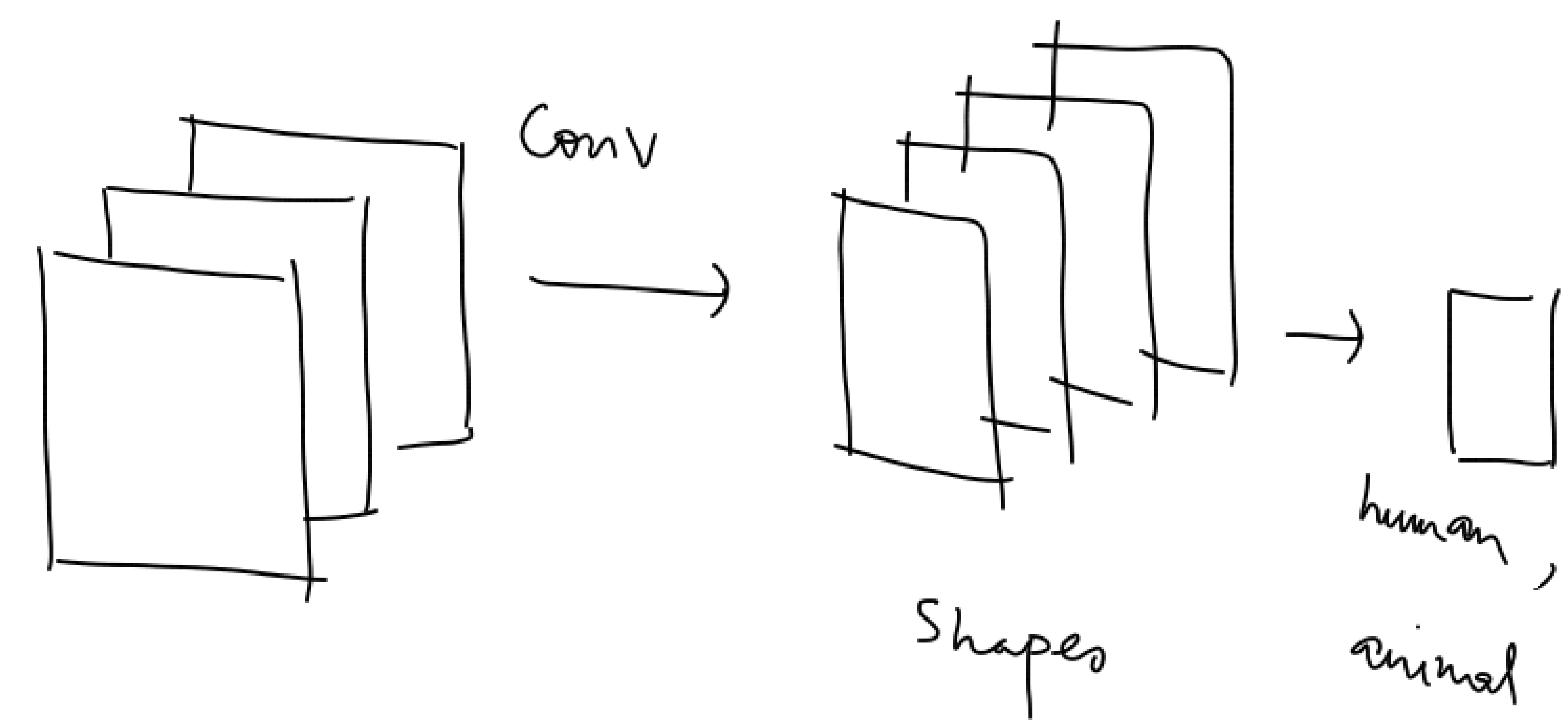
Lec 14 Convolutional Neural Networks





Heart : Convolution operation

obj : reduce image size without compromising the information content



$$f \circledast g(x) = \int_{-\infty}^{+\infty} f(t)\, g(x-t)\, dt$$ wiki

Signal processing

image

1	20	20	20	1
20	1	1	1	20
20	1	1	1	20
1	20	1	20	1
1	1	20	1	1

$\boxed{n \times n}$

5×5

$\circledast$

filter

1	0	-1
1	0	-1
1	0	-1

3×3

$f \times f$

Vertical edge detector

=

filtered image

21	0	-21
38	0	-38
0	0	0

$(n-f+1) \times (n-f+1)$

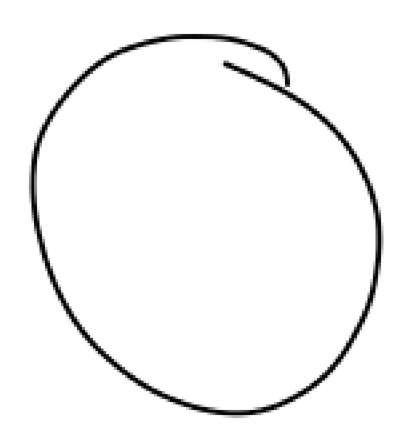
filters

find shapes / useful information

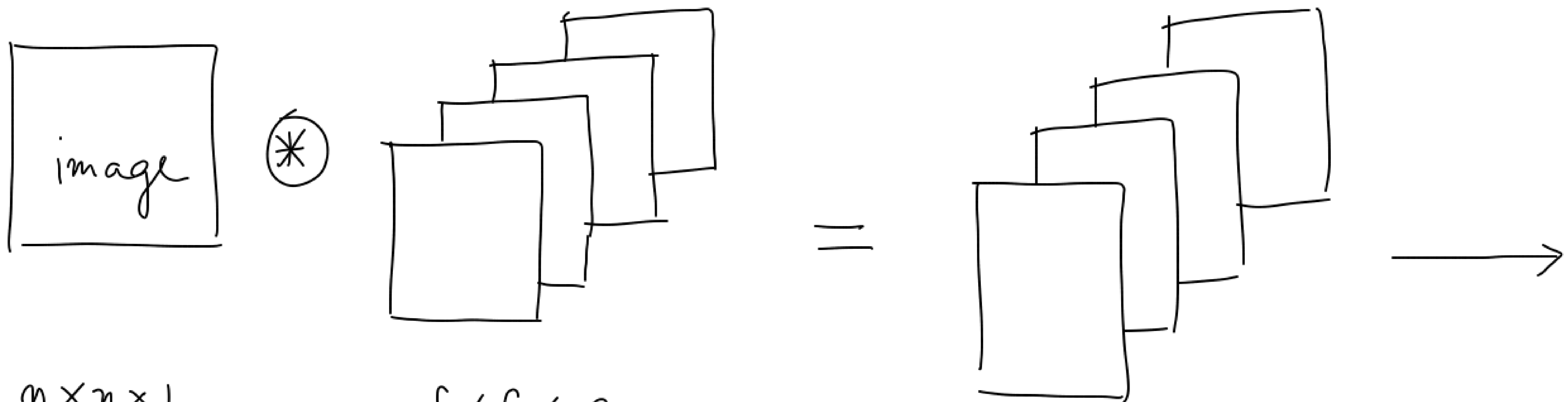
[0 ———— 255]

0 Black

255 White

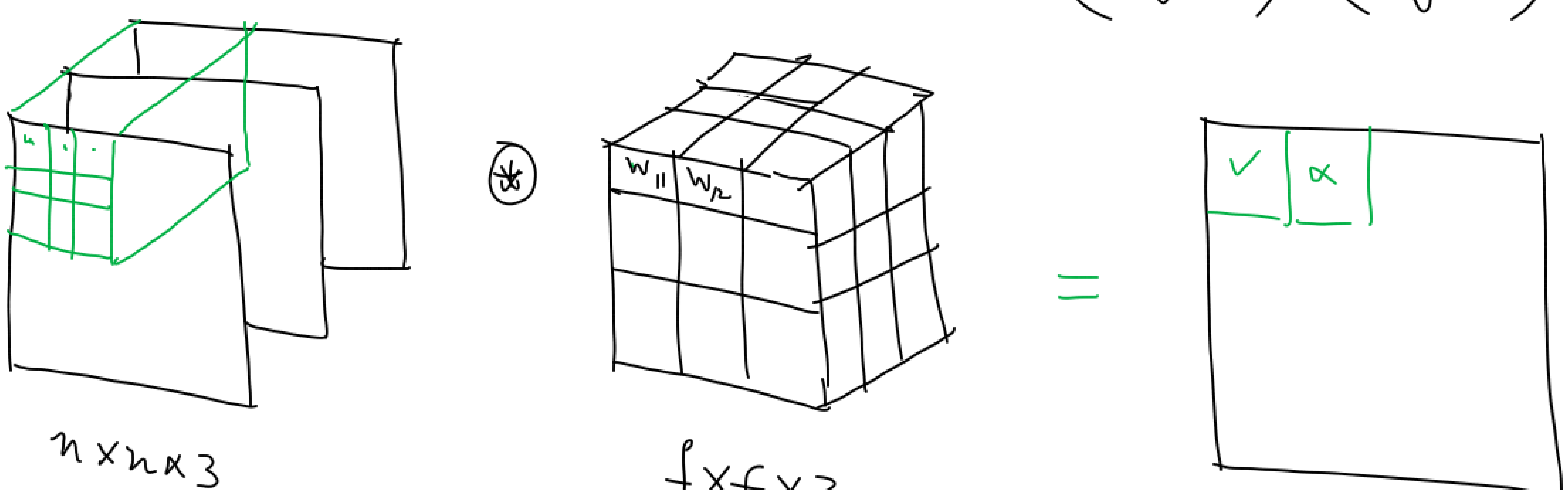


$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \longrightarrow \text{horizontal}$$



$n \times n \times 1$ ⊛ $f \times f \times c$ = $(n-f+1) \times (n-f+1) \times c$ →

- face
- hand
- .
- .

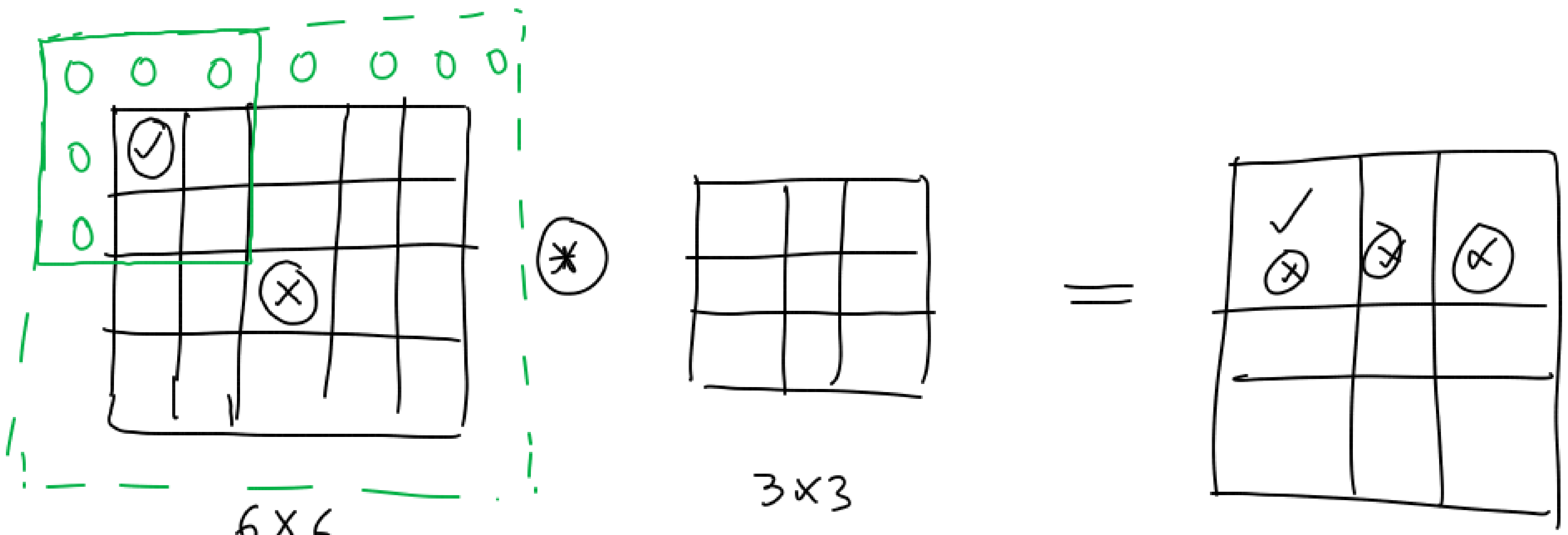


$n \times n \times 3$ ⊛ $f \times f \times 3$ = $(n-f+1) \times (n-f+1)$

Q: How to find these filters?

A: training the weights of filters.

Padding



6X6

$(n+2) \times (n+2)$

padded image

8X8

3x3

$f \times f$

6X6

$n' = n + 2p$

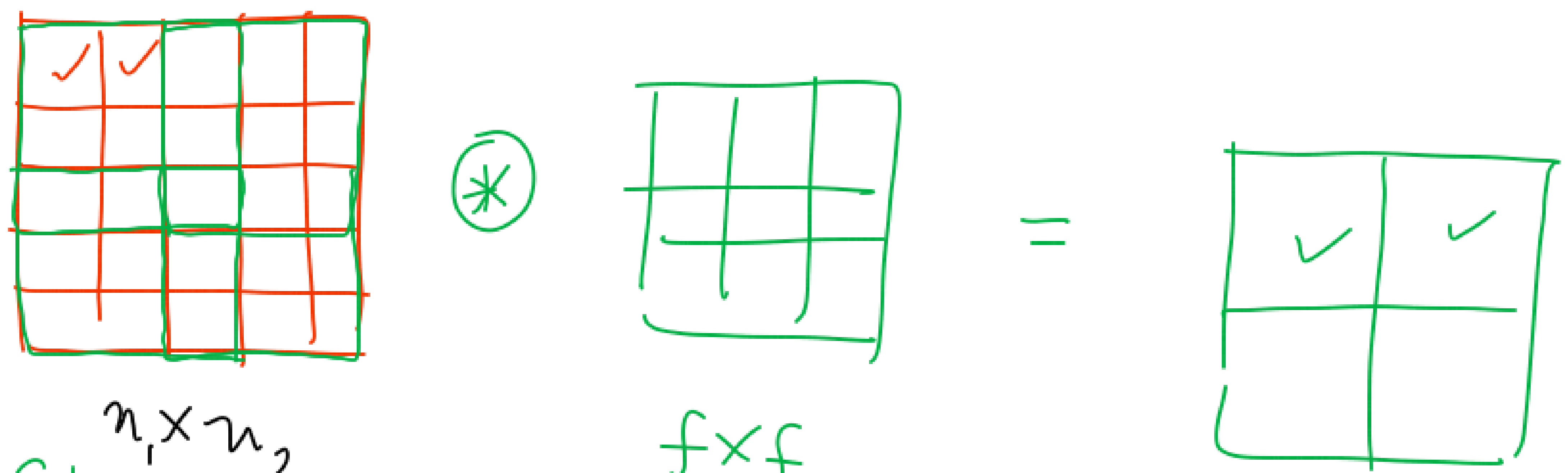
$(n' - f + 1) = n$

$\Rightarrow \quad p = \dfrac{f-1}{2}$, f is odd.

"VALID" convolution – no padding

"SAME" convolution – padding s.t. the final image size is the same as original given the filter.

Stride : ✓



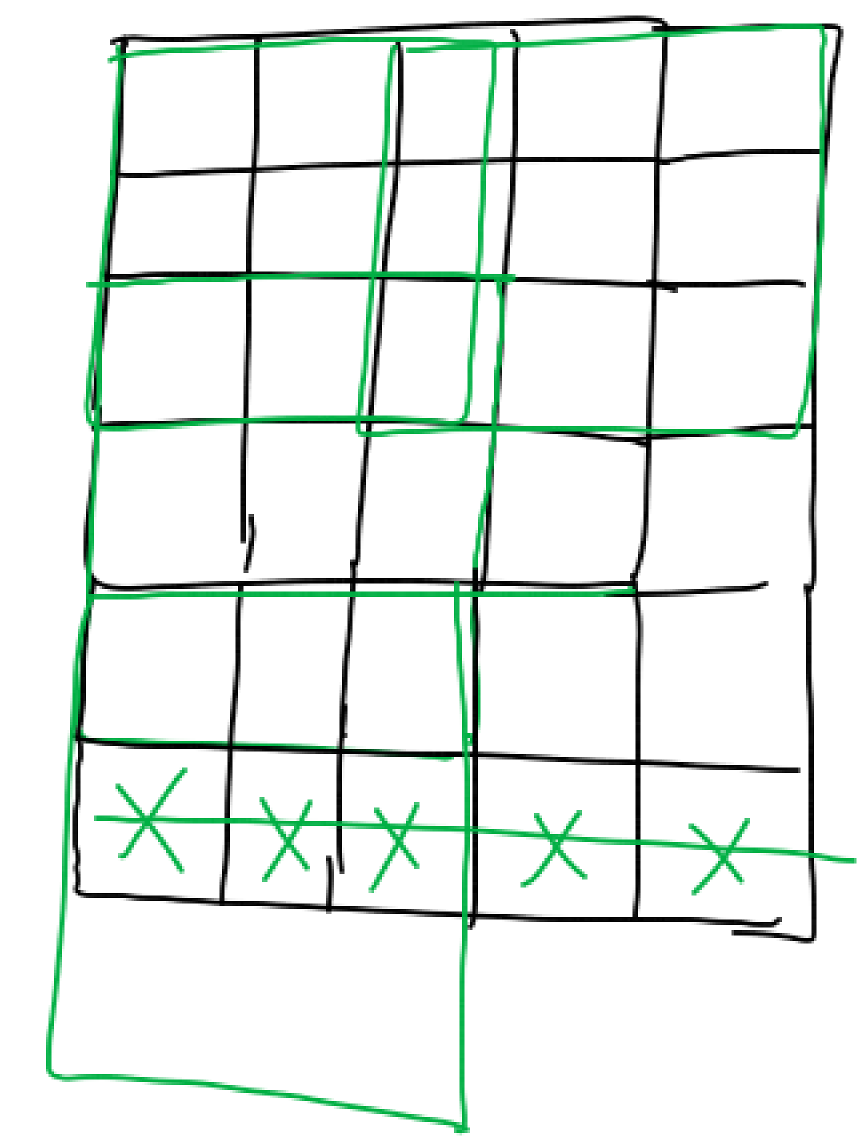
$n_1 \times n_2$ $f \times f$

Stride

$\boxed{\text{stride} = 2}$

Standard, stride = 1.

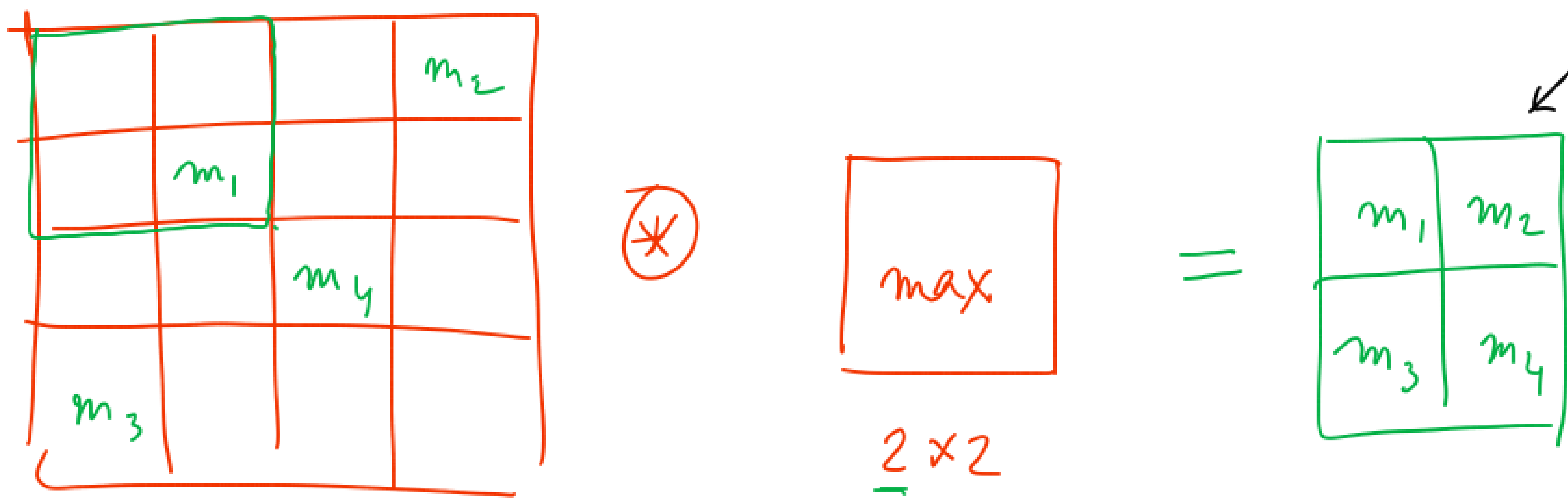
stride = s



$$\left\lfloor \frac{n_1 - f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n_2 - f}{s} + 1 \right\rfloor$$

stride ignores the part of the image where the convolution can't be entirely within the image.

Max pooling layer:



2 × 2

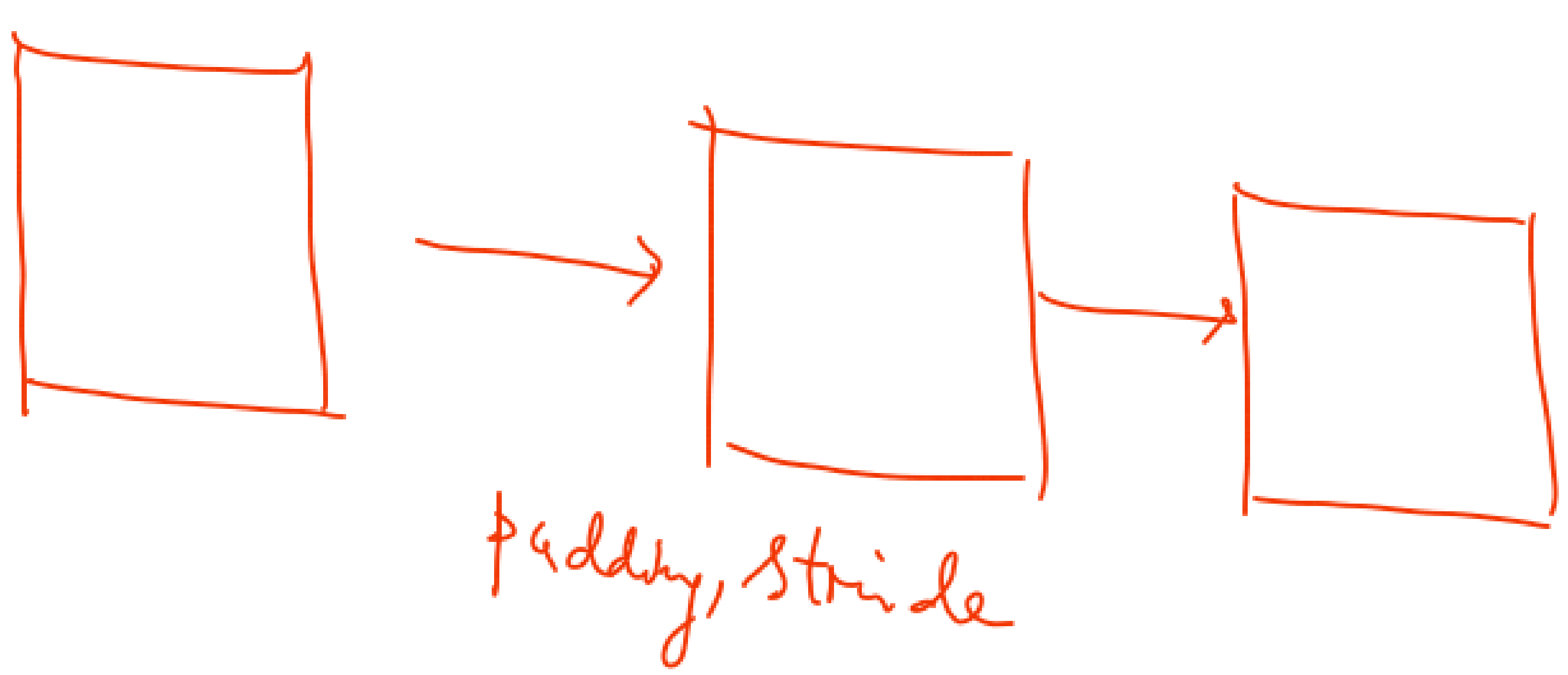
stride = 2

reducing size

Sharpening / Contrast increase

Other alternatives:

Average pooling

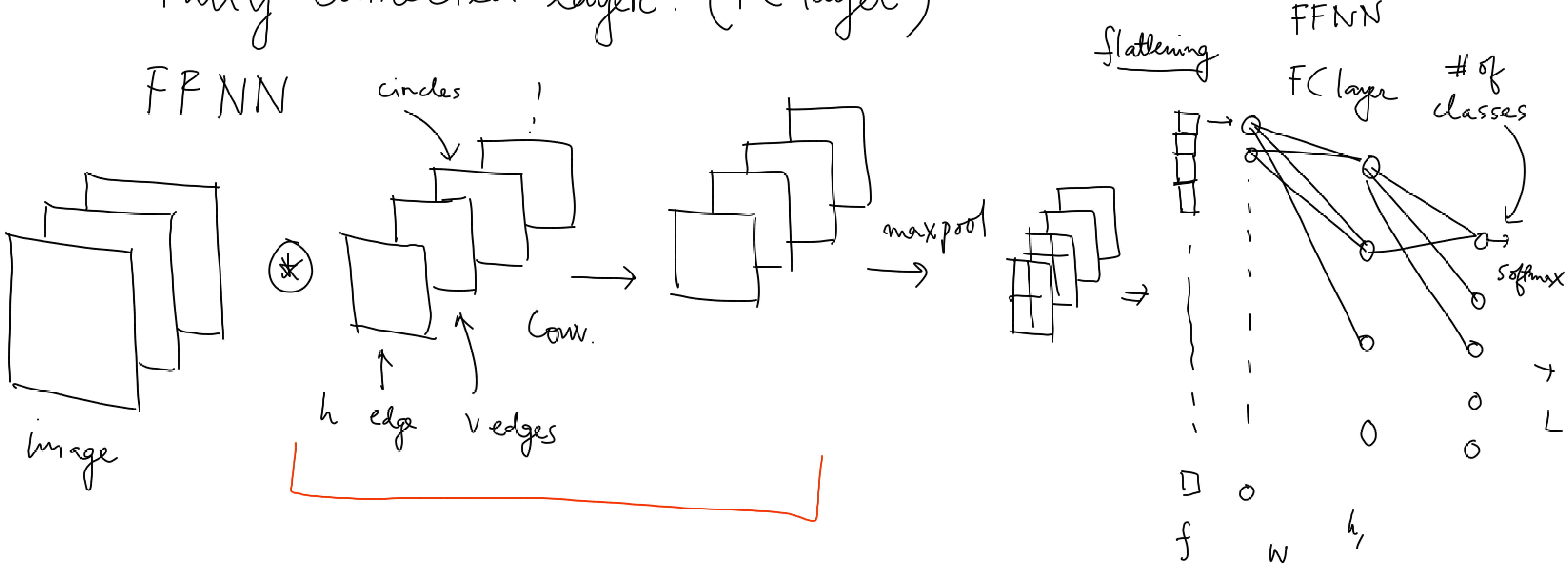


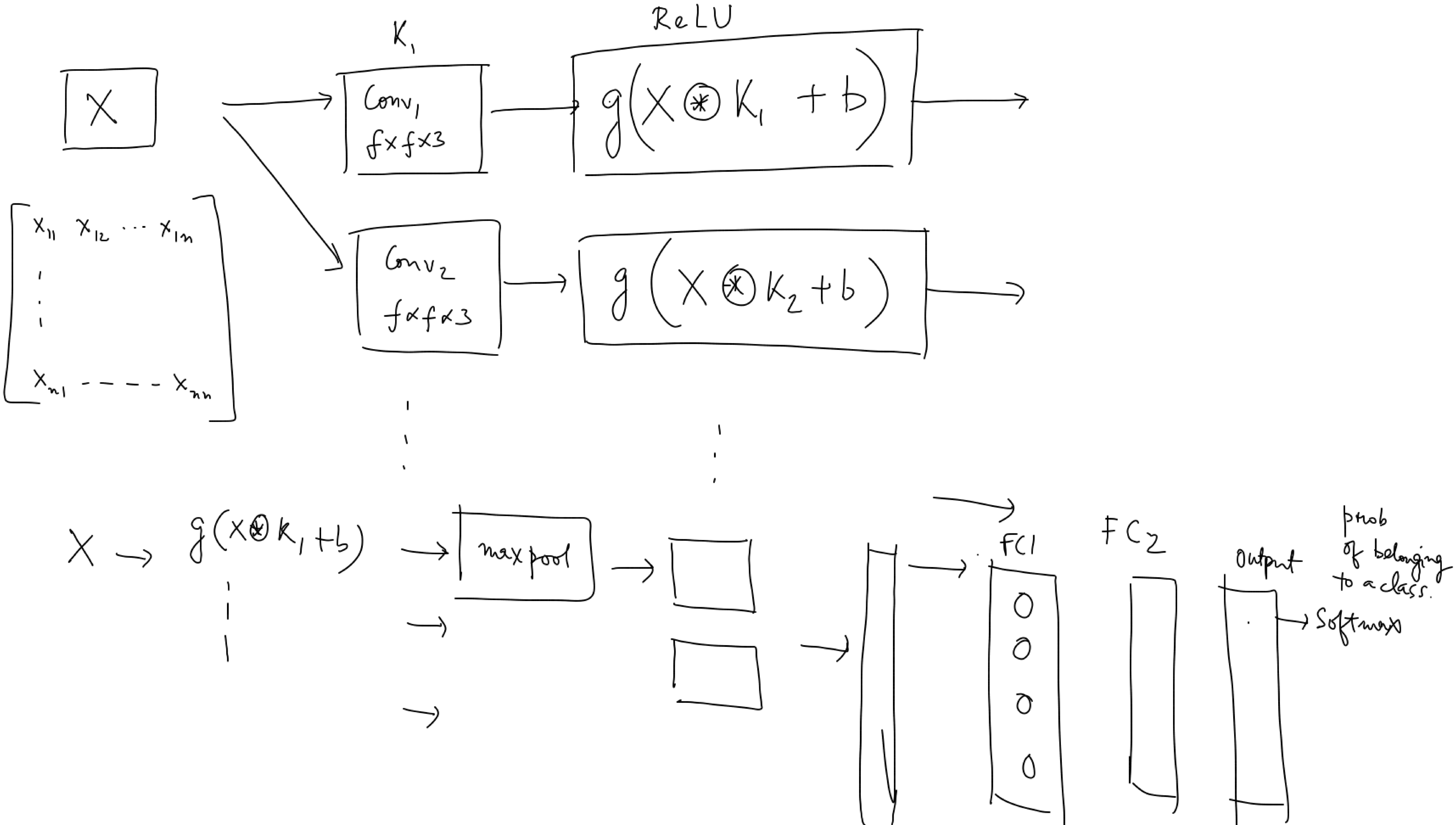
typically strides = side of the filter.
size

Conv. layers extract features from images

Q: How to use those features to classify the images?

A: Fully connected layer. (FC layer)





X
x_{11} x_{12} ... x_{1n}
x_{n1} ---- x_{nn}
K_1
Conv$_1$
f×f×3
ReLU
$g(X \circledast K_1 + b)$
Conv$_2$
f×f×3
$g(X \circledast K_2 + b)$
X
$g(X \circledast K_1 + b)$
max pool
FC1
FC2
output
prob of belonging to a class.
Softmax

$$\boxed{X} \xrightarrow{K,\,B} \boxed{Z} \longrightarrow \boxed{g} \longrightarrow g(z) \longrightarrow \boxed{f} \xrightarrow[y_1]{} \boxed{FC} \longrightarrow L$$

FFNN

$$\frac{\partial L}{\partial y}$$

$$\boxed{X} \circledast \boxed{K} + B$$

filter/kernel

$$K^{(t+1)} \leftarrow K^{(t)} - \eta \left.\frac{\partial L}{\partial K}\right|_t$$

$$B^{(t+1)} \leftarrow B^{(t)} - \eta \left.\frac{\partial L}{\partial B}\right|_t$$

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix} \circledast \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} + B = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$Z_{11} = X_{11} \cdot K_{11} + X_{12} K_{12} + X_{21} K_{21} + X_{22} K_{22} + B$$

$$Z_{12} =$$

$$Z_{22} =$$

$$\frac{\partial L}{\partial K_{mn}} = \sum_{i,j} \frac{\partial L}{\partial Z_{ij}} \frac{\partial Z_{ij}}{\partial K_{mn}}$$

$$\frac{\partial L}{\partial K_{11}} = \frac{\partial L}{\partial Z_{11}} \cdot \underbrace{\frac{\partial Z_{11}}{\partial K_{11}}}_{X_{11}} + \frac{\partial L}{\partial Z_{12}} \cdot \underbrace{\frac{\partial Z_{12}}{\partial K_{11}}}_{X_{12}} + \frac{\partial L}{\partial Z_{21}} \cdot \underbrace{\frac{\partial Z_{21}}{\partial K_{11}}}_{X_{21}} + \frac{\partial L}{\partial Z_{22}} \cdot \underbrace{\frac{\partial Z_{22}}{\partial K_{11}}}_{X_{22}}$$

HW: Verify $\Rightarrow$ $\underline{\dfrac{\partial L}{\partial K}} = X \circledast \dfrac{\partial L}{\partial Z}$ --- (1)

$$\frac{\partial L}{\partial B} = \sum_{ij} \frac{\partial L}{\partial Z_{ij}}$$

$$\frac{\partial L}{\partial X_{mn}} = \sum_{ij} \frac{\partial L}{\partial Z_{ij}} \cdot \frac{\partial Z_{ij}}{\partial X_{mn}} \longrightarrow$$

HW: $\underline{\dfrac{\partial L}{\partial X}} = \left(\text{zeropadded } \dfrac{\partial L}{\partial Z} \right) \circledast$ (180° clockwise rotated K)

$$K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

180° clockwise

$$= \begin{bmatrix} K_{22} & K_{21} \\ K_{12} & K_{11} \end{bmatrix}$$