Lec 20: Sequential and Simultaneous move Games

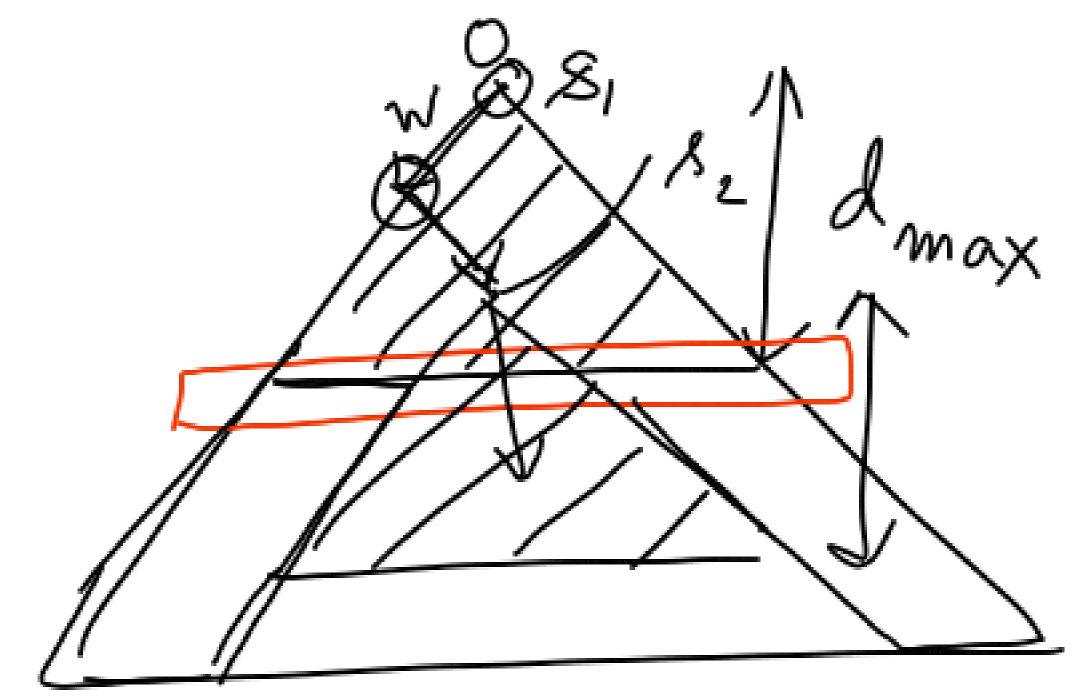
Backward induction:

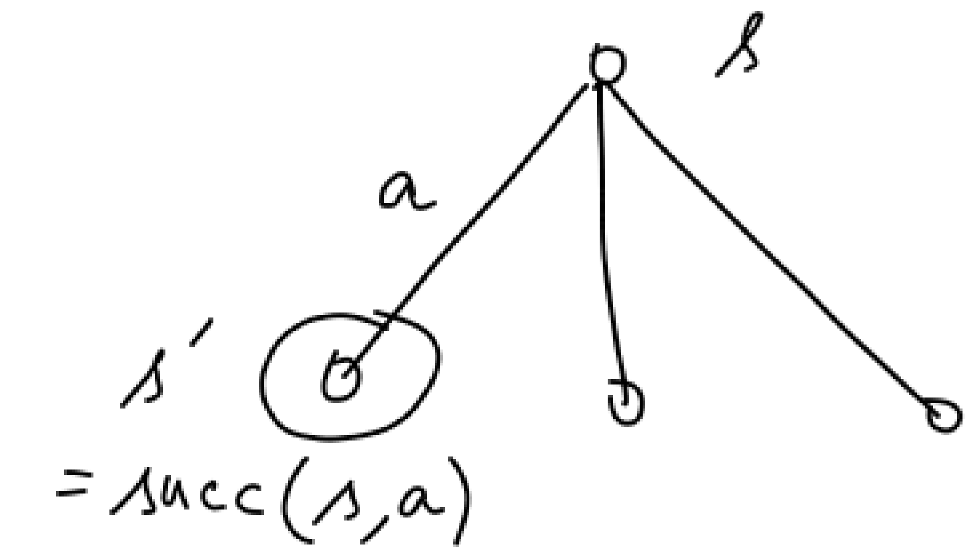
limitation: Computation for large games

10^{40}, 10^{170}

Speedup methods

1 Depth-limited search





$$U_{agent}(s,d) = \begin{cases} utility(s) & \text{if } isEnd(s) \\ eval(s) & \text{if } d=0 \\ \max\limits_{a \in actions(s)} U_{agent}(succ(s,a), d-1) & \text{if } player(s) = agent \\ \min\limits_{a \in actions(s)} U_{agent}(succ(s,a), d-1) & \text{if } player(s) = opp \end{cases}$$

depth

eval(s) is a domain specific function denoting the possible utility to the agent

In chess

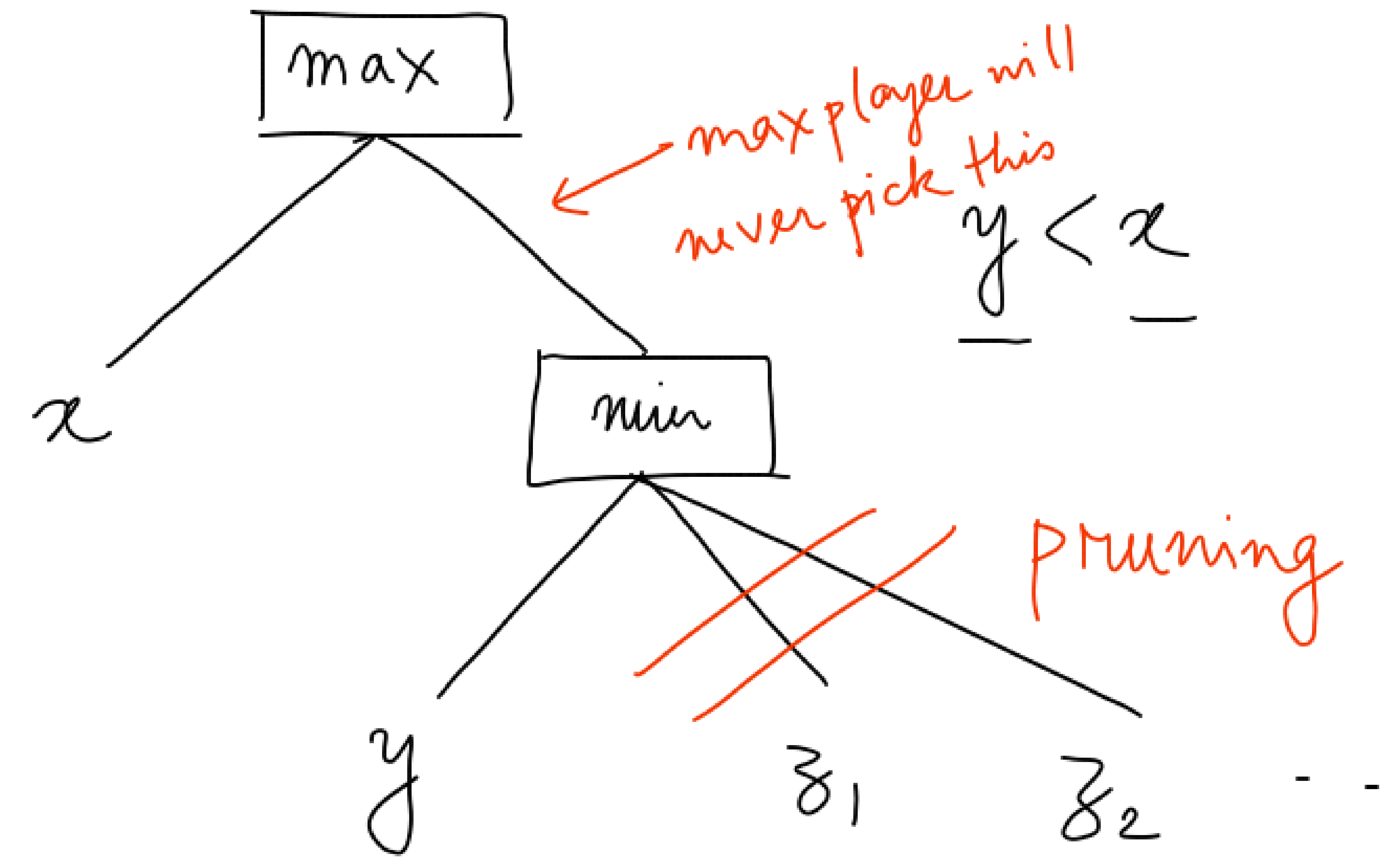
$$\text{eval}(s) = \text{army} + \text{mobility} + \text{king-safety} + \dots$$

$$\begin{cases} \text{army} = \underline{10^{100}}(K - K') + 9(Q - Q') + \underline{5}(R - R') + \dots \\ \text{mobility} = c \times \#\text{ of } (\text{legal moves} - \text{legal moves}') \end{cases}$$

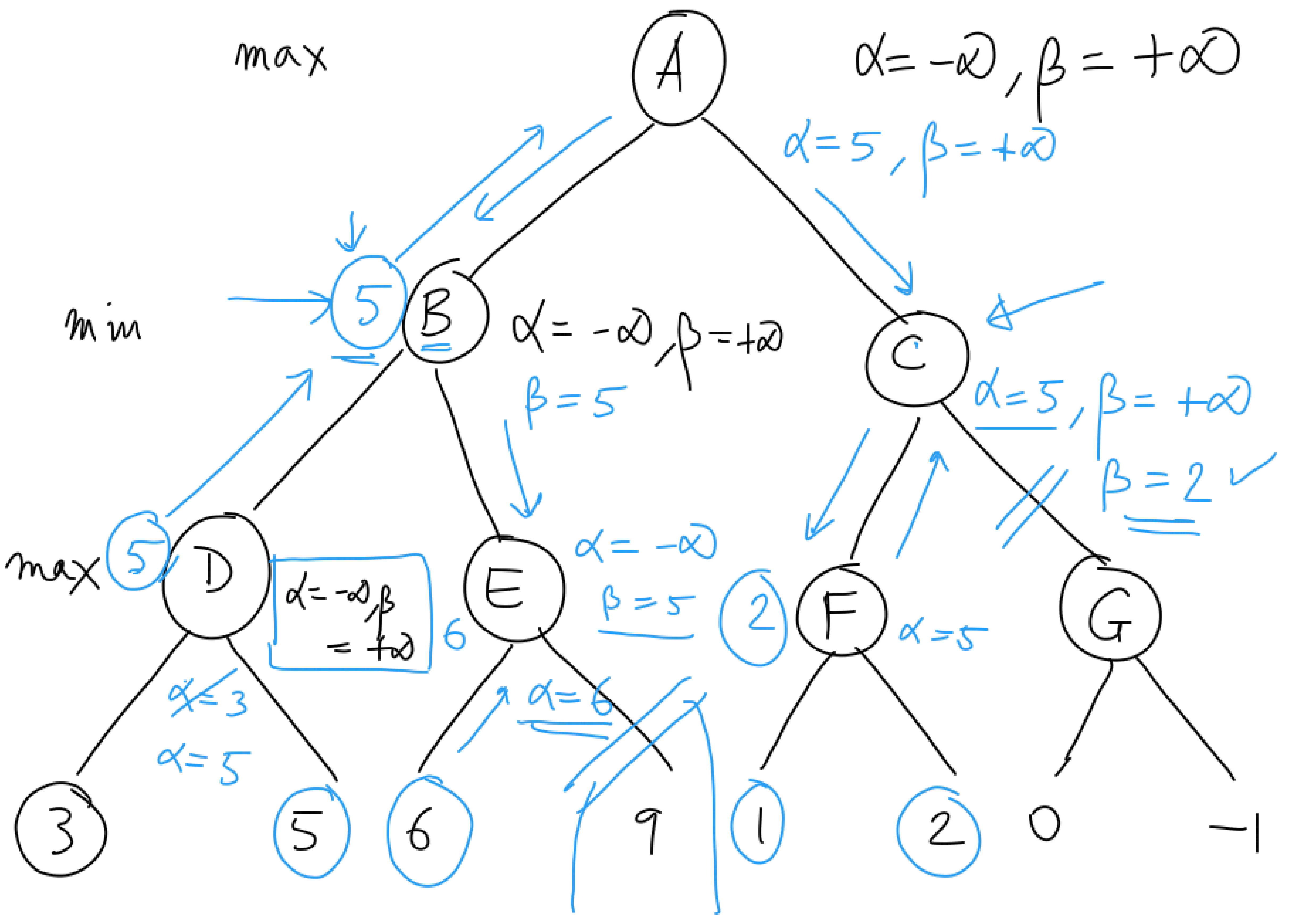
$$\boxed{u_{agent}(s, d_{max})}$$

heuristic

② Pruning (α-β pruning)



Stockfish 16



max nodes

min nodes

$\alpha \rightarrow \max\{\alpha, \text{value encountered}\}$

$\beta \rightarrow \min\{\beta, \text{value encountered}\}$

$\alpha \geq \beta$?

Simultaneous move games

		Goalkeeper		
		L	C	R
Shooter	L	$\cancel{1}$ -1	1	1
	C	1	-1	1
	R	1	1	-1

~~awesome left side shooter~~

matrix game → two player zero sum simultaneous move games.

(L, L) → an simultaneous move equilibrium

Equilibrium: A tuple of actions from which no player gains by an unilateral deviation.

gains → $>$ current utility

unilateral ↓ other players remain at the same tuple of actions → only the concerned player is moving.

		2: L	C	R	min
1	L	1	1	1	1
	C	1	−1	1	−1
	R	1	1	−1	−1
max		1	1	1	

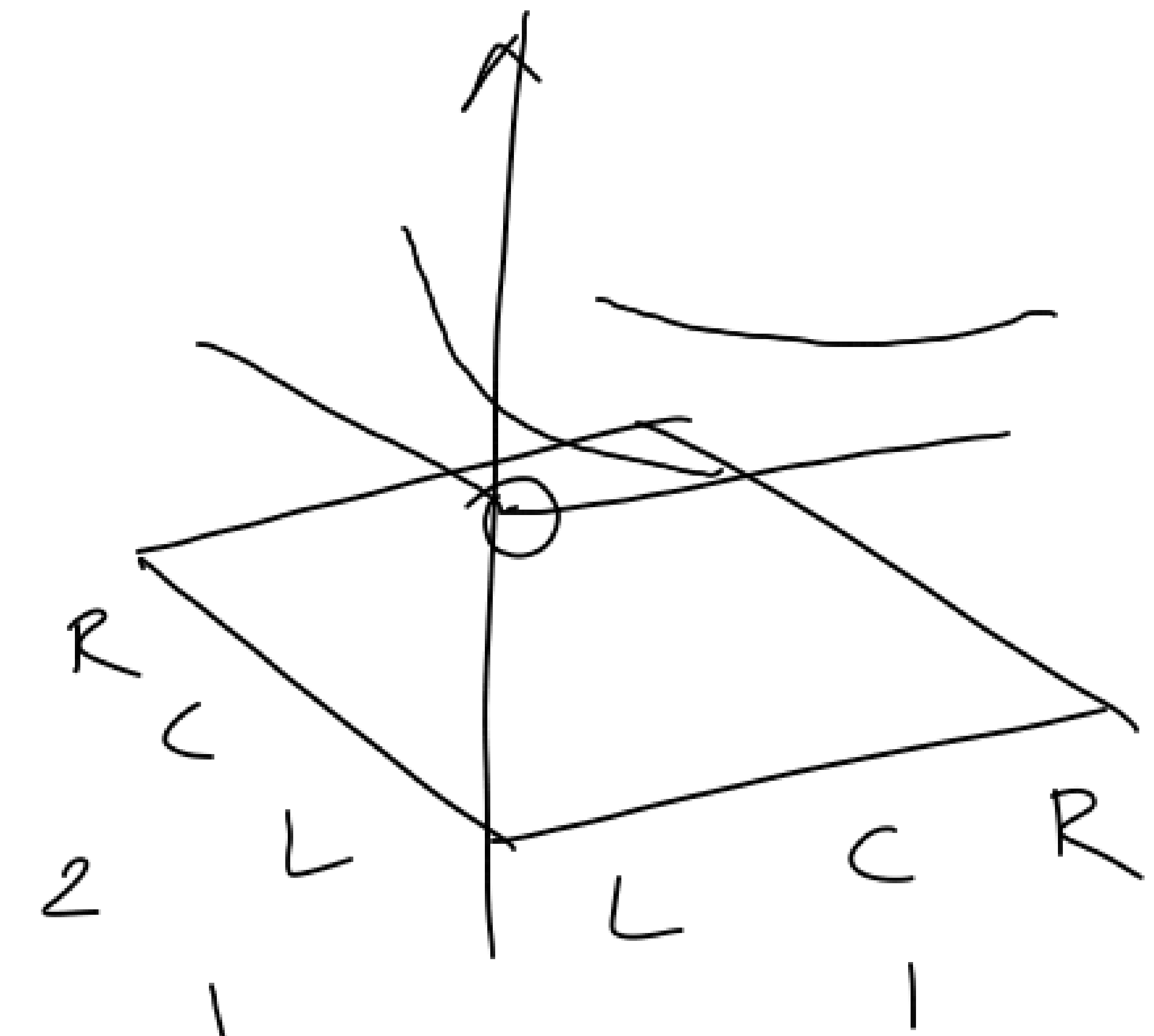
$\underbrace{1, -1, -1}_{\text{max min} = 1}$

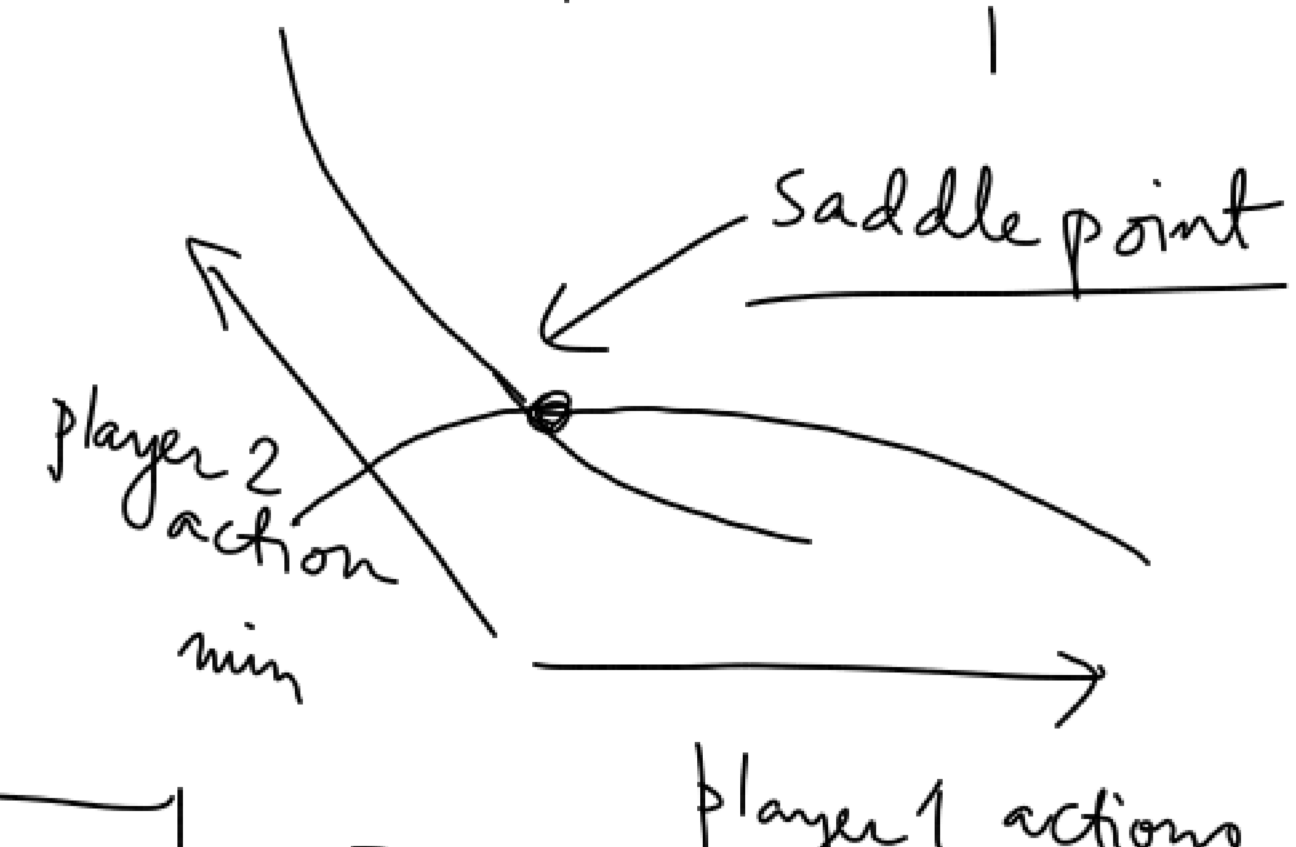
$\underbrace{1 \quad 1 \quad 1}_{\text{min max} = 1}$

			min
−1	1	1	−1
1	−1	1	−1
1	1	−1	−1
max: 1	1	1	

$\underbrace{-1, -1, -1}_{-1}$

$\underbrace{1 \quad 1 \quad 1}_{1}$





Saddle point exists/not?

$$\max_{s_1} \min_{s_2} u(s_1, s_2) = \underline{v}$$

L, GR

$$\min_{s_2} \max_{s_1} u(s_1, s_2) = \overline{v}$$

Which one is larger?

- $\overline{v} \geq \underline{v}$?
- $\overline{v} \leq \underline{v}$?
- incomparable?

$$u(s_1, s_2) \geq \min_{s_2} u(s_1, s_2) \quad \underline{\forall s_1, s_2}$$

$$\min_{s_2} u(s_1^*, s_2) = \max_{s_1} \underbrace{\min_{s_2} u(s_1, s_2)}_{f(s_1)}$$

$$\max_{s_1} u(s_1, s_2) \geq u(s_1^*, s_2) \geq$$

$$\hookrightarrow \min_{s_2} \max_{s_1} u(s_1, s_2) \geq \max_{s_1} \min_{s_2} u(s_1, s_2)$$

$$\boxed{\overline{v} \geq \underline{v}}$$