Quick recap: Game Theory

- Analytical approach for predicting reasonable outcome
- Building blocks: players, strategies, utilities
- Difference between action and strategy
- Key assumptions: rationality and intelligence

Example to illustrate: Game of Chess (von Neumann and Morgenstern, 1944)

Formal description

- Two player game: White and Black – 16 pieces each.
- Every piece has some legal moves – ACTIONS
- Starts with W, players take turns
- Ends: W win, if W captures B king
  B win, if B captures W king
  Draw, if nobody has legal moves but kings are not in check, both players agree to a draw, board position is such that nobody can win, many more ...

Natural questions from a theorist's perspective

- Does W have a winning strategy, i.e., a plan of moves s.t. it wins IRRESPECTIVE of the moves of B?
- Does B have a winning strategy?
- Or at least guarantee a draw?
- Neither may be possible – not synonymous with end of game.

What is a strategy?

In the context of chess,

board position if different from Game Situation

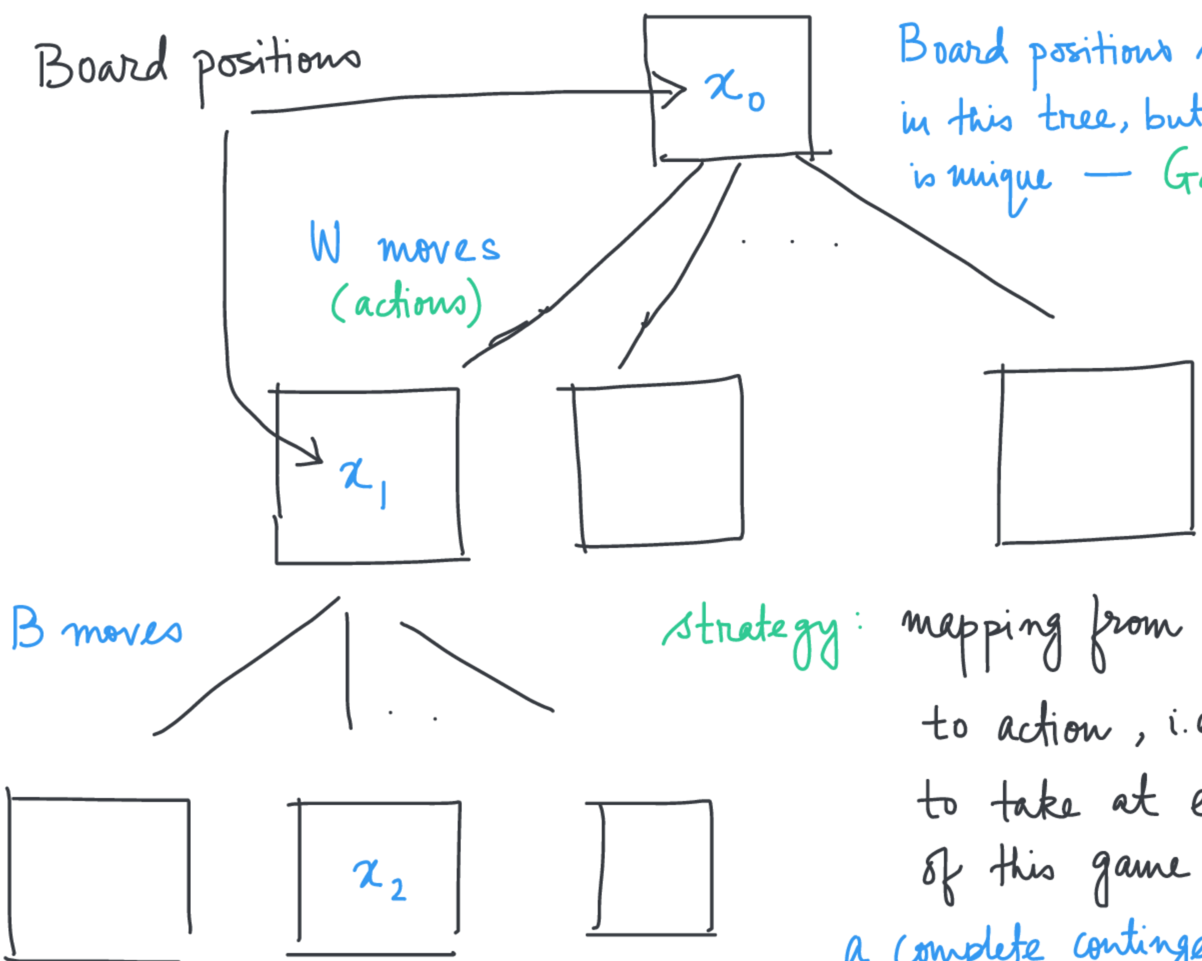
more than one sequence of moves can bring to the same board position.

denote a board position by x_k

Game Situation is a finite sequence $(x_0, x_1, x_2, \dots, x_K)$ of board positions s.t.

- x_0 is the opening board position
- $x_k \to x_{k+1}$, k even – created by a single action of W

  k odd – created by a single action of B



Board positions may repeat in this tree, but a vertex is unique — Game Situation

strategy: mapping from game situation to action, i.e. what action to take at every vertex of this game tree.

a complete contingency plan

A strategy for W is a function s_W that associates every game situation $(x_0, x_1, \ldots, x_K) \in H$ (set of all game situations), K even, with a board position x_{K+1} such that the move $x_K \to x_{K+1}$ is a single valid move of W.

Similar definition of s_B for B.

Note:

- strategy pair (s_W, s_B) determines $\boxed{\text{an outcome}}$ also called one play of the game. - a path through the game tree

Questions: (1) this is a finite game - where does it end?

(2) can a player guarantee an outcome?

The game ends: (a) W wins OR (b) B wins OR (c) Draw.

A winning strategy for W is a strategy s_W^* s.t. for every s_B (s_W^*, s_B) ends in a win for W.

A strategy guaranteeing at least a draw for W is s_W' s.t. for every s_B, (s_W', s_B) either ends in a draw or win for W.

analogous definitions of s_B^* and s_B'

Not obvious if such strategies exist

An early result of Game Theory (Von Neumann, 1928)

In chess, one and only one of the following statements is true

1. W has a winning strategy
2. B has a winning strategy
3. Each player has a strategy guaranteeing a draw

- there were other possibilities, e.g., nothing can be guaranteed
- it does not say what is that strategy

  actually it is not known: which one is true and what is that strategy

Chess would have been a boring game if any of these answers were known.