Normal Form Games

It is a representation technique for games

$N = \{1, 2, \dots, n\}$ set of players

S_i : set of strategies of player i, $s_i \in S_i$

Set of strategy profiles $S = \underset{i \in N}{\times} S_i$

A strategy profile $s = (s_1, s_2, \dots, s_n) \in S$

strategy profile without i

$$\underline{s}_{-i} = (s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_n)$$

$u_i : S \to \mathbb{R}$ utility function of player i

NFG representation is the tuple $\langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$

If S_i is finite $\forall i \in N$, this is called a finite game.

Example : Penalty Kick Game

Goalkeeper

Shooter	L	C	R
L	-1,1	1,-1	1,-1
C	1,-1	-1,1	1,-1
R	1,-1	1,-1	-1,1

$N = \{1, 2\}$

$S_1 = S_2 = \{L, C, R\}$

$u_1(L, L) = -1$, $u_1(L, C) = 1$,

$u_1(L, R) = 1$

$u_2(L, L) = 1$, $u_2(L, C) = -1$

$u_2(L, R) = -1$

Rationality: A player is rational if she picks actions to maximize her utility

Intelligence: A player is intelligent if she knows the rules of the game perfectly and picks action considering that there are other rational and intelligent players.

Common Knowledge:

A fact is common knowledge if

1. all players know the fact
2. all players know that all players know the fact
3. all players know that all other players know that all other players know the fact

... ad infinitum

Implication:

- Isolated island: three blue-eyed people (eye color can be blue or black) no reflecting medium on the island, nobody talks about eye color
- One day a sage comes to the island and says

  Blue-eyed people are bad for the island and must leave. There is at least one blue-eyed person in this island.

sage cannot be disputed - if someone realizes that his/her eye color is blue he/she leaves at the end of the day.

How does common knowledge percolate?

If there were only one blue-eyed person, he would see the other two person has black eyes. Sage is always correct, hence he must be the only blue-eyed person — leaves at end of day 1.

If there were two, each of them would see one blue, one black. Watch the other blue-eyed person's move till day 2 (since the other blue-eyed person also knows that fact). When the other person doesn't leave by day 1, both are certain about their eye-color and leaves at the end of day 2. The black-eyed person watches this till day 3 and does not leave.

Since there are 3 people with blue eyes, all of them leaves on day 3.

Assumption: The fact that all players are rational and intelligent is a common knowledge.