Domination in NFGs

Players 1 \ 2	L	M	R
U	1, 0	1, 3	3, 2
D	−1, 6	0, 5	3, 3

Strategies of player 2

Strategies of Player 1

Will a rational player ever play R?

Dominated Strategy

A strategy $s_i' \in S_i$ of player i is **strictly dominated** if there exists another strategy s_i of i such that **for every strategy profile** $s_{-i} \in S_{-i}$ of the other players $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$.

A strategy $s_i' \in S_i$ of player i is **weakly dominated** if there exists another strategy s_i of i such that **for every strategy profile** $s_{-i} \in S_{-i}$ of the other players $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$, and

there exists some $\tilde{s}_{-i} \in S_{-i}$ such that

$$u_i(s_i, \tilde{s}_{-i}) > u_i(s_i', \tilde{s}_{-i}).$$

Example: R is strictly dominated, D is weakly dominated.

Dominant Strategy

A strategy s_i is strictly (weakly) dominant strategy for player i if s_i strictly (weakly) dominates all other $s_i' \in S_i \setminus \{s_i\}$

Examples: ① Neighboring Kingdoms' dilemma

	Agri	Defence
Agri	5, 5	0, 6
Defence	6, 0	1, 1

Dominant strategy?

Which kind?

② One indivisible item for sale

Two players having values v_1 and v_2 respectively

Each player can choose a number in $[0, M]$, $(M \gg v_1, v_2)$

Player quoting the largest number "wins" the object (tie broken in favor of 1), and "pays" the losing player's chosen number

utility of winning player = her value − her payment

utility of losing player = 0

NFG representation: $N = \{1, 2\}$, $S_1 = S_2 = [0, M]$

$$u_1(s_1, s_2) = \begin{cases} v_1 - s_2, & \text{if } s_1 \geq s_2 \\ 0 & \text{ow} \end{cases} \qquad u_2(s_1, s_2) = \begin{cases} v_2 - s_1, & \text{if } s_1 < s_2 \\ 0 & \text{ow} \end{cases}$$

Dominant strategy? Which kind?

Dominant Strategy Equilibrium

A strategy profile $(s_1^*, s_2^*, \cdots, s_n^*)$ is a strictly (weakly) dominant strategy equilibrium (SDSE/WDSE) if s_i^* is a strictly (weakly) dominant strategy for i, $\forall i \in N$.

Question: What kind of equilibrium in this game?

	D	E
A	5, 5	0, 5
B	5, 0	1, 1
C	4, 0	1, 1