Risk aversion of Players

Risk : if the other player does not pick the equilibrium action

Less risky for player 1 : T.

1 \ 2	L	R
T	2,1	1,-20
M	3,0	-10,1
B	-100,2	3,3

Another type of rationality : players making pessimistic estimates of others

This worst case optimal choice is max-min strategy

$$s_i^{maxmin} \in \arg\max_{s_i \in S_i} \min_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$$

maxmin value

$$\underline{v}_i = \max_{s_i \in S_i} \min_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$$

$$u_i(s_i^{maxmin}, t_{-i}) \geq \underline{v}_i \ , \ \forall t_{-i} \in S_{-i}$$

Max-min and dominant strategies

<u>Theorem</u>: If s_i^* is a dominant strategy for player i, then it is a maxmin strategy for i.

Proof outline [for strictly dominant strategies]

Let s_i^* be the strictly dominant strategy of player i

(a) $$u_i(s_i^*, s_{-i}) > u_i(s_i', s_{-i}), \quad \forall s_{-i} \in S_{-i}, \forall s_i' \in S_i \setminus \{s_i^*\}$$

let $s_{-i}^{\min}(s_i') \in \arg\min_{s_{-i} \in S_{-i}} u_i(s_i', s_{-i})$ - worst choice of the other players for i.

but (a) holds for all s_{-i}

$$u_i(s_i^*, s_{-i}^{\min}(s_i')) > u_i(s_i', s_{-i}^{\min}(s_i')), \quad \forall s_i' \in S_i \setminus \{s_i^*\}$$

$$s_i^* \in \arg\max_{s_i \in S_i} \min_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i}).$$

Weak dominance : Homework.

Relationship with PSNE

Every PSNE $s^* = (s_1^*, \ldots, s_n^*)$ of an NFG satisfies

$$u_i(s^*) \geq \underline{v}_i, \quad \forall i \in N.$$

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Proof:

$$u_i(s_i, s_{-i}^*) \geq \min_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i})$$ [by defn. of min]

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \forall s_i \in S_i$$ [by defn of PSNE]

$$u_i(s_i^*, s_{-i}^*) = \max_{s_i \in S_i} u_i(s_i, s_{-i}^*) \geq \max_{s_i \in S_i} \min_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i}) = \underline{v}_i$$