Recap: ① dominance cannot explain all reasonable outcomes

② PSNE - unilateral deviation [STABILITY]

③ Maxmin - rationality for risk-aversion [SECURITY]

What happens to stability and security when some strategies are eliminated?

Iterated elimination of dominated strategies

	L	C	R
T	1,2	2,3	0,3
M	2,2	2,1	3,2
B	2,0	0,0	1,0

Order: T, R, B, C → (M,L): 2,2

Order: B, L, C, T → (M,R): 3,2

Does it change the maxmin value?

Consider the example above:

		Pl 1	Pl 2
	maxmin	2	0
B is eliminated (dominated for 1)	maxmin	2	2

Maxmin value is not affected for the player whose dominated strategy is removed

Theorem: Consider NFG $G = \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$, let $\hat{s}_j \in S_j$ be a dominated strategy. Let $\hat{G}$ be the residual game after removing $\hat{s}_j$. The maxmin value of j in $\hat{G}$ is equal to her maxmin value in G.

Intuition: maxmin is the max of the mins - elimination affects one min but that doesn't affect the max since the strategy was dominated.

Proof: maxmin value of j in G, $\underline{v}_j = \max_{s_j \in S_j} \min_{\underline{s}_{-j} \in \underline{S}_{-j}} u_j(s_j, \underline{s}_{-j})$

maxmin value of j in $\hat{G}$, $\underline{\hat{v}}_j = \max_{s_j \in S_j \setminus \{\hat{s}_j\}} \min_{\underline{s}_{-j} \in \underline{S}_{-j}} u_j(s_j, \underline{s}_{-j})$

let t_j dominates $\hat{s}_j$ in G, $t_j \in S_j \setminus \{\hat{s}_j\}$

$$u_j(t_j, \underline{s}_{-j}) \geqslant u_j(\hat{s}_j, \underline{s}_{-j}), \quad \forall \underline{s}_{-j} \in \underline{S}_{-j}$$

Therefore,

$$\min_{\underline{s}_{-j} \in \underline{S}_{-j}} u_j(t_j, \underline{s}_{-j}) = u_j(t_j, \underline{\tilde{s}}_{-j}) \geqslant u_j(\hat{s}_j, \underline{\tilde{s}}_{-j}) \geqslant \min_{\underline{s}_{-j} \in \underline{S}_{-j}} u_j(\hat{s}_j, \underline{s}_{-j})$$

$$\Rightarrow \max_{s_j \in S_j \setminus \{\hat{s}_j\}} \min_{\underline{s}_{-j} \in \underline{S}_{-j}} u_j(s_j, \underline{s}_{-j}) \geqslant \min_{\underline{s}_{-j} \in \underline{S}_{-j}} u_j(t_j, \underline{s}_{-j}) \geqslant \min_{\underline{s}_{-j} \in \underline{S}_{-j}} u_j(\hat{s}_j, \underline{s}_{-j})$$

$\underline{v}_j$ [max min value in G for j]

$$= \max_{s_j \in S_j} \min_{s_{-j} \in S_{-j}} u_j(s_j, s_{-j})$$

$S_j \setminus \{\hat{s}_j\}$ $\quad$ $\hat{s}_j$

$$= \max \left\{ \max_{s_j \in S_j \setminus \{\hat{s}_j\}} \min_{s_{-j} \in S_{-j}} \cdots \quad , \min_{s_{-j} \in S_{-j}} u_j(\hat{s}_j, s_{-j}) \right\}$$

$\geq$

$$= \max_{s_j \in S_j \setminus \{\hat{s}_j\}} \min_{s_{-j} \in S_{-j}} u_j(s_j, s_{-j}) = \hat{\underline{v}}_j$$ [max min of j in $\hat{G}$]