Recap: ① iterated elimination of dominated strategies
② Preservation of equilibrium
③ stability & security coincide for matrix games
④ limited to pure strategies – PSNE may not exist

	L	R
L	-1,1	1,-1
R	1,-1	-1,1

Mixed strategies

probability distribution over the set of strategies

		$4/5$	$1/5$
		L	R
$2/3$	L	-1,1	1,-1
$1/3$	R	1,-1	-1,1

Consider a finite set A

define $\Delta A = \{ p \in [0,1]^{|A|} : \sum_{a \in A} p_a = 1 \}$

set of all probability distributions over A.

σ_i is a mixed strategy of player i

$\sigma_i \in \Delta(S_i)$, i.e., $\sigma_i : S_i \to [0,1]$ s.t. $\sum_{s_i \in S_i} \sigma_i(s_i) = 1$.

We are discussing non-cooperative games, the players choose their strategies independently

The joint probability of 1 picking s_1 and 2 picking $s_2 = \sigma_1(s_1)\,\sigma_2(s_2)$

utility of player i at a mixed strategy profile (σ_i, σ_{-i}) is

$$u_i(\sigma_i, \sigma_{-i}) = \sum_{s_1 \in S_1} \sum_{s_2 \in S_2} \cdots \sum_{s_n \in S_n} \sigma_1(s_1)\,\sigma_2(s_2) \cdots \sigma_n(s_n)\; u_i(s_1, s_2, \ldots, s_n)$$

we are overloading u_i to denote the utility at pure and mixed strategies.

Utility at a mixed strategy is the expectation of the utilities at pure strategies.

So, all the rules of expectation holds, e.g., linearity.

Example:

		$4/5$	$1/5$
		L	R
$2/3$	L	$-1, 1$	$1, -1$
$1/3$	R	$1, -1$	$-1, 1$

$$u_1(\sigma_1, \sigma_2) = \frac{2}{3} \cdot \frac{4}{5} \cdot (-1) + \frac{2}{3} \cdot \frac{1}{5} \cdot 1 + \frac{1}{3} \cdot \frac{4}{5} \cdot 1 + \frac{1}{3} \cdot \frac{1}{5} \cdot (-1)$$

mixture of mixed strategies

$$u_i(\lambda \sigma_i + (1-\lambda)\sigma_i', \sigma_{-i}) = \lambda u_i(\sigma_i, \sigma_{-i}) + (1-\lambda)\, u_i(\sigma_i', \sigma_{-i}).$$