How to find an MSNE?

Support of mixed strategy (prob. distribution)

For mixed strategy σ_i, the subset of strategy space of i on which σ_i has positive mass is the support of σ_i, i.e.,

$$\delta(\sigma_i) = \{ s_i \in S_i : \sigma_i(s_i) > 0 \}$$

using the definition of support, here is a characterization of MSNE

<u>Theorem</u>: A mixed strategy profile $(\sigma_i^*, \sigma_{-i}^*)$ is a MSNE iff $\forall i \in N$

1. $u_i(s_i, \sigma_{-i}^*)$ is the same for all $s_i \in \delta(\sigma_i^*)$
2. $u_i(s_i, \sigma_{-i}^*) \geq u_i(s_i', \sigma_{-i}^*)$, $\forall s_i \in \delta(\sigma_i^*), s_i' \notin \delta(\sigma_i^*)$

<u>Implication</u>: consider penalty shootout game

	L	R
L	-1,1	1,-1
R	1,-1	-1,1

Case 1: Supports $(\{L\}, \{L\})$

for player 1, $s_1' = R$ violates 2

Case 2: $(\{L, R\}, \{L\})$ - symmetric for the other case

for player 1, the expected utility has to be same for L and R - not possible - violates 1

Case 3: $(\{L,R\}, \{L,R\})$

(2) is vacuously satisfied

for (1), player 1 chooses L w.p. p and player 2 chooses L w.p. q

(1) for player 1

$$u_1(L, (q, 1-q)) = u_1(R, (q, 1-q))$$

$$(-1)\,q + 1\cdot(1-q) = 1\cdot q + (-1)\,(1-q)$$

$$\Rightarrow \quad q = 1/2$$

	L	R
L	-1,1	1,-1
R	1,-1	-1,1

(1) for player 2

$$u_2((p, 1-p), L) = u_2((p, 1-p), R) \quad \Rightarrow \quad p = 1/2$$

$$\text{MSNE} = \left(\left(\tfrac{1}{2}, \tfrac{1}{2}\right), \left(\tfrac{1}{2}, \tfrac{1}{2}\right)\right)$$

Exercises:

	F	C
F	2,1	0,0
C	0,0	1,2

	F	C	D
F	2,1	0,0	1,1
C	0,0	1,2	2,0