Strategies in IIEFGs

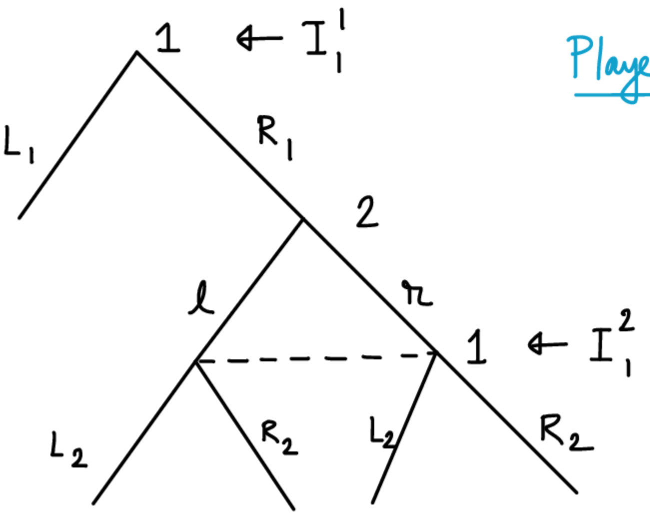
Strategy set of i : $S_i = \bigtimes_{j=1}^{k(i)} X(I_i^j)$

Randomized strategies in IIEFG

In NFGs, mixed strategies randomize over pure strategies

In EFGs, randomization can happen in different ways

- randomize over the strategies defined at the beginning of the game
- randomize over the action at an information set - behavioral strategy



Player 1

Pure strategies at the beginning

$(L_1 L_2), (L_1 R_2), (R_1 L_2), (R_1 R_2)$

mixed strategy σ_1

$\sigma_1(L_1 L_2), \sigma_1(L_1 R_2), \sigma_1(R_1 L_2), \sigma_1(R_1 R_2)$

---

actions at I_1^1 : L_1, R_1 ;

at I_1^2 : L_2, R_2

behavioral strategies

$b_1(I_1^1) \in \Delta(L_1, R_1)$

$b_1(I_1^2) \in \Delta(L_2, R_2)$

Definition: Behavioral Strategy

A behavioral strategy of a player in an IIEFG is a function that maps each of her information sets to a probability distribution over the set of possible actions at that information set.

Question: What is the relation between mixed and behavioral strategies?

In this example: MSs live in $\mathbb{R}^4$, BSs live in two $\mathbb{R}^2$ spaces mixed strategies look a "richer" or "larger" concept

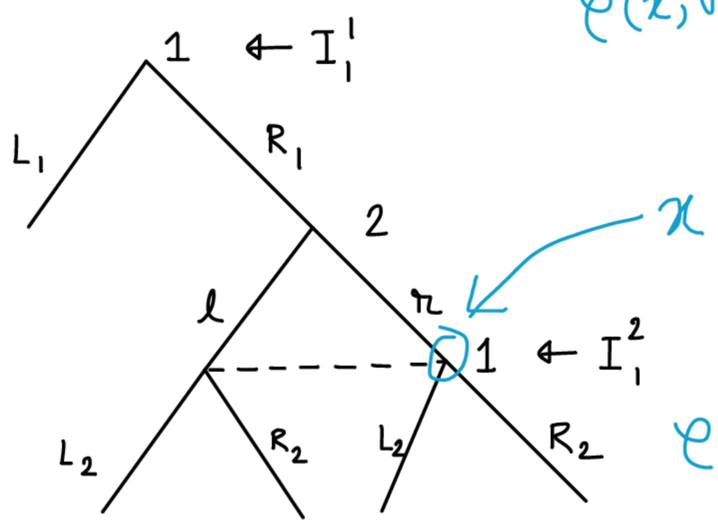
Can a player attain higher payoff in one strategy than the other?

Question: Can we have an equivalence?

Equivalence in terms of the probability of reaching a vertex/history x

Say $\rho(x;\sigma)$ is the probability of reaching node x under mixed strategy profile σ.

Similarly, $\rho(x;b)$ is the same for behavioral strategy profile b.



$$\rho(x;\sigma) = \sigma_1(R_1)\,\sigma_2(r)$$
$$= \big(\sigma_1(R_1L_2) + \sigma_1(R_1R_2)\big)\cdot \sigma_2(r)$$

$$\rho(x;b) = b_1(I_1^1)(R_1)\cdot b_2(I_2^1)(r)$$

Important: different players can choose different kind of strategies

e.g., if 1 chooses σ_1 above and 2 chooses b_2 then

$$\rho(x;\sigma_1,b_2) = \big(\sigma_1(R_1L_2) + \sigma_1(R_1R_2)\big)\cdot b_2(I_2^1)(r)$$

Definition: equivalence

A mixed strategy σ_i and a behavioral strategy b_i of a player i in an IIEFG are equivalent if every mixed/behavioral strategy vector ξ_{-i} of the other players and every vertex x in the game tree

$$\rho(x;\sigma_i,\xi_{-i}) = \rho(x;b_i,\xi_{-i})$$

Example: in the game above

$$b_1(I_1^1)(L_1) = \sigma_1(L_1 L_2) + \sigma_1(L_1 R_2)$$
$$b_1(I_1^1)(R_1) = \sigma_1(R_1 L_2) + \sigma_1(R_1 R_2)$$

b_1 and σ_1 are equivalent

$$b_1(I_2^1)(L_2) = \sigma_1(L_2 \mid R_1)$$
$$b_1(I_2^1)(R_2) = \sigma_1(R_2 \mid R_1)$$

equivalent strategies induce same probability of reaching a vertex

More on equivalent strategies

The equivalence, by definition, holds at the leaf nodes too

Claim: it is enough to check the equivalence only at the leaf nodes

Reason: pick an arbitrary non-leaf node, the probability of reaching that node is equal to the sum of the probabilities of reaching the leaf nodes in its subtree.

This argument can be extended further

Theorem (Utility equivalence)

If σ_i and b_i are equivalent, then for every mixed/behavioral strategy vector of the other players ξ_{-i}, the following holds

$$u_j(\sigma_i, \xi_{-i}) = u_j(b_i, \xi_{-i}), \ \forall j \in N.$$

Repeat the argument for any equivalent mixed and behavioral str profiles

Corollary: Let σ and b are equivalent, i.e., σ_i and b_i are equivalent $\forall i \in N$.

Then $u_i(\sigma) = u_i(b)$.