Relationship between DSI and DSIC

Revelation principle (for DSI SCFs): If there exists an indirect mechanism that implements f in dominant strategies, then f is DSIC.

Implication: can focus on DSIC mechanisms WLOG.

Proof: Let f is implemented by $\langle M_1, \ldots, M_n, g \rangle$, hence $\exists\, s_i : \Theta_i \to M_i$ s.t. $\forall i \in N, \forall\, \tilde{\underline{m}}_{-i}, m_i', \theta_i$,

$$u_i\left(g(s_i(\theta_i), \tilde{\underline{m}}_{-i}), \theta_i\right) \geqslant u_i\left(g(m_i', \tilde{\underline{m}}_{-i}), \theta_i\right) \quad \text{—— (1)}$$

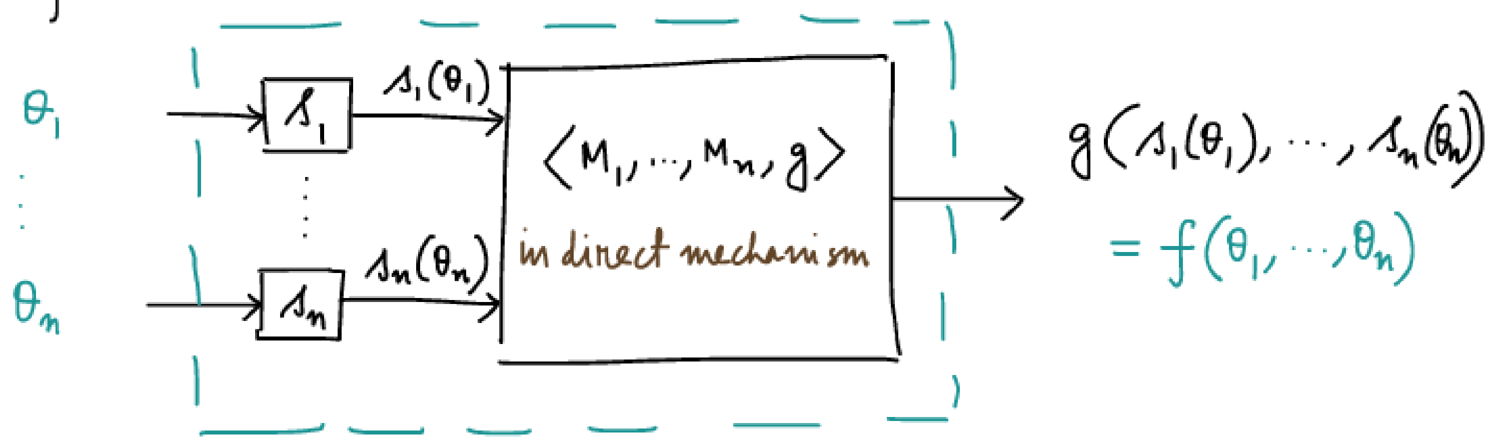
and
$$g(s_i(\theta_i), \underline{s}_{-i}(\underline{\theta}_{-i})) = f(\theta_i, \underline{\theta}_{-i}) \quad \text{—— (2)}$$

Eqn. (1) holds for all $m_i', \tilde{\underline{m}}_{-i}$, in particular, $m_i' = s_i(\theta_i')$, $\tilde{\underline{m}}_{-i} = \underline{s}_{-i}(\tilde{\underline{\theta}}_{-i})$ where θ_i' and $\tilde{\underline{\theta}}_{-i}$ are arbitrary. Hence

$$u_i\big(\underbrace{g(s_i(\theta_i), \underline{s}_{-i}(\tilde{\underline{\theta}}_{-i}))}_{= f(\theta_i, \tilde{\underline{\theta}}_{-i})}, \theta_i\big) \geqslant u_i\big(\underbrace{g(s_i(\theta_i'), \underline{s}_{-i}(\tilde{\underline{\theta}}_{-i}))}_{= f(\theta_i', \tilde{\underline{\theta}}_{-i})}, \theta_i\big) \quad [\text{By } (2)]$$

$$\Rightarrow u_i\left(f(\theta_i, \tilde{\underline{\theta}}_{-i}), \theta_i\right) \geqslant u_i\left(f(\theta_i', \tilde{\underline{\theta}}_{-i}), \theta_i\right)$$

$\Rightarrow$ f is DSIC.



$\langle \Theta_1, \ldots, \Theta_n, f \rangle$, direct mechanism

Bayesian extension (agents may have probabilistic information about others' types)

Types are generated from a common prior (common knowledge) and are revealed only to the respective agents.

Recall: Bayesian games

$$\langle N, (M_i)_{i\in N}, (\Theta_i)_{i\in N}, P, (\Gamma_\theta)_{\theta\in\Theta} \rangle$$

taking the place of actions

$s_i : \Theta_i \to M_i$, message mapping

Defn. An (indirect) mechanism $\langle M_1, \dots, M_n, g \rangle$ implements an SCF f in Bayesian equilibrium if

① $\exists$ a message mapping profile $(s_1, \dots, s_n)$, s.t., $s_i(\theta_i)$ maximizes the ex-interim utility of agent i, $\forall \theta_i, \forall i \in N$, i.e.,

$$\mathbb{E}_{\underline{\theta}_{-i}|\theta_i}[u_i(g(s_i(\theta_i), \underline{s}_{-i}(\underline{\theta}_{-i})), \theta_i)] \geq \mathbb{E}_{\underline{\theta}_{-i}|\theta_i}[u_i(g(m_i', \underline{s}_{-i}(\underline{\theta}_{-i})), \theta_i)]$$

$\forall m_i', \forall \theta_i, \forall i \in N$, and

② $g(s_i(\theta_i), \underline{s}_{-i}(\underline{\theta}_{-i})) = f(\theta_i, \underline{\theta}_{-i}), \forall \theta$.

We call f is Bayesian implementable via $\langle M_1, \dots, M_n, g \rangle$ under the prior P.

Lemma: If an SCF f is dominant strategy implementable, then it is Bayesian implementable.

Proof: homework.

A direct mechanism $\langle \Theta_1, \ldots, \Theta_n, f \rangle$ is Bayesian Incentive Compatible (BIC) if $\forall \theta_i, \theta_i', \forall i \in N$

$$\mathbb{E}_{\underline{\theta}_{-i}|\theta_i}\left[u_i(f(\theta_i, \underline{\theta}_{-i}), \theta_i)\right] \geq \mathbb{E}_{\underline{\theta}_{-i}|\theta_i}\left[u_i(f(\theta_i', \underline{\theta}_{-i}), \theta_i)\right].$$

Revelation principle (for BI SCFs)

If an SCF f is implementable in Bayesian equilibrium, then f is BIC.

Proof idea is similar to the DSI, with expected utilities at appropriate places.

For truthfulness of these two kinds, we will only consider incentive compatibility.

These results hold even for ordinal preferences and mechanisms.