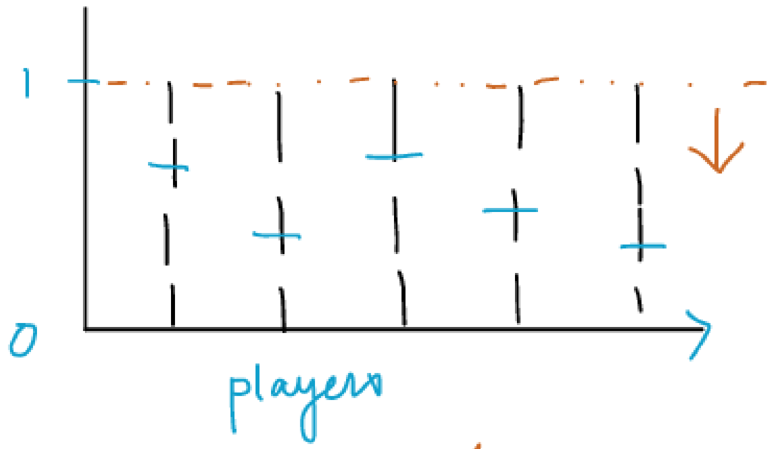
How to ensure PE, ANON, and SP in task allocation domain?

Uniform rule (Sprumont 1991)



Suppose, $\sum_{i \in N} p_i < 1$

Begin with everyone's allocation being 1. Keep reducing until $\sum f_i = 1$

Whenever some agent's peak is reached, set the allocation for that agent to be its peak

Definition:

① $f_i^u(P) = p_i$ if $\sum_{i \in N} p_i = 1$

② $f_i^u(P) = \max\{p_i, \mu(P)\}$ if $\sum_{i \in N} p_i < 1$

where $\mu(P)$ solves $\sum_{i \in N} \max\{p_i, \mu\} = 1$.

③ $f_i^u(P) = \min\{p_i, \lambda(P)\}$ if $\sum_{i \in N} p_i > 1$

where $\lambda(P)$ solves $\sum_{i \in N} \min\{p_i, \lambda\} = 1$.

Q: Is this ANON, PE, and SP?

Theorem (Sprumont 1991)

The uniform rule SCF is ANON, PE, and SP.

Proof: ANON is obvious - only the peaks matter and not their owners.

PE: the allocation is s.t.

$$f_i^u(P) = p_i, \forall i \in N, \text{ if } \sum p_i = 1$$

$$f_i^u(P) \geq p_i, \forall i \in N, \text{ if } \sum p_i < 1$$

$$f_i^u(P) \leq p_i, \forall i \in N, \text{ if } \sum p_i > 1$$

for some players the peaks are allocated, and for others the allocation is the same. This is PE, since any other allocation can only improve the allocation of a player at the cost of another player's allocation.

Strategyproofness.

for case $\sum p_i = 1$, every agent gets their peak - no reason to deviate.

Case $\sum p_i < 1$, then $f_i^u(P) \geq p_i \; \forall i \in N$.

only possible manipulation for agents that have $f_i^u(P) > p_i$

$\Rightarrow \mu(P) > p_i$, i.e., the allocation stopped before reaching p_i. The only way i can change the allocation is by reporting $p_i' > \mu(P) > p_i$ — but this is a worse allocation for i than $\mu(P)$.

Similar argument for case $\sum p_i > 1$. This completes the proof.

The converse is also true. We skip the proof.

Thm: An SCF is SP, PE, and ANON iff it is the uniform rule.

Ref: Sprumont (1991) : Division problem with single peaked preferences.