Mechanism Design with Transfers

Social Choice Function $F : \Theta \to X$

X : space of all outcomes

In this domain, an outcome x has two components allocation a and payment vector $\pi = (\pi_1, \dots, \pi_n)$, $\pi_i \in \mathbb{R}$

Examples of allocations

① A public decision of building a bridge, park, or museum.

$a \in A = \{\text{park}, \text{bridge}, \dots\}$

② Allocation of a divisible good, e.g., a shared spectrum

$a = (a_1, a_2, \dots, a_n)$, $a_i \in [0,1]$, $\sum_{i \in N} a_i = 1$

a_i : fraction of the resource i gets.

③ Single indivisible object allocation

$a = (a_1, \dots, a_n)$, $a_i \in \{0,1\}$, $\sum_{i \in N} a_i \leq 1$

④ Partition of indivisible objects.

S = set of objects

$A = \{(A_1, \dots, A_n) : A_i \subseteq S \ \forall i \in N, A_i \cap A_j = \phi \ \forall i \neq j\}$

Type of an agent i is $\theta_i \in \Theta_i$, this is a private information of i.

Agents' benefit from an allocation is defined via the valuation function

Valuation depends on the allocation and type of the player

$v_i : A \times \Theta_i \to \mathbb{R}$ [independent private values]

E.g., if i has a type "environment saver" θ_i^{env}

and $a \in \{\text{Bridge}, \text{Park}\}$, $v_i(B, \theta_i^{env}) < v_i(P, \theta_i^{env})$

the value can change if the type changes to "business friendly" θ_i^{bus}

$$v_i(B, \theta_i^{bus}) > v_i(P, \theta_i^{bus})$$

Payments $\pi_i \in \mathbb{R}$, $\forall i \in N$

Payment vector $\underline{\pi} = (\pi_1, \pi_2, \dots, \pi_n)$

Utility of player i, when its type is θ_i and the outcome is $(a, \underline{\pi})$ is given by $u_i((a, \underline{\pi}), \theta_i) = v_i(a, \theta_i) - \pi_i$

possibly non-linear

linear in payment

Quasi-linear payoffs

Q: Why is this a domain restriction?

Consider two alternatives (a, π) and (a, π')

Suppose $\pi_i' < \pi_i$, there cannot be any preference profile in the quasi-linear domain where (a, π) is more preferred than (a, π') for agent i.

The utilities are $v_i(a, \theta_i) - \pi_i' > v_i(a, \theta_i) - \pi_i$

In the complete domain both orders would have been feasible.

This simple restriction opens up the opportunity for a lot of SCFs to satisfy interesting properties