The Vickrey-Clarke-Groves Mechanism (VCG)

The most popular mechanism in the Groves class

Also known as the pivotal mechanism (V'61, C'71, G'73)

Given by a unique $h_i(\underline{\theta}_{-i})$ function

$$h_i(\underline{\theta}_{-i}) = \max_{a \in A} \sum_{j \neq i} v_j(a, \theta_j)$$

The payment is modified to

$$p_i^{VCG}(\theta_i, \underline{\theta}_{-i}) = \max_{a \in A} \sum_{j \neq i} v_j(a, \theta_j) - \sum_{j \neq i} v_j(f^{eff}(\theta_i, \underline{\theta}_{-i}), \theta_j)$$

Note: $p_i^{VCG}(\theta) \not\geq 0 \quad \forall \theta \in \Theta, \forall i \in N$ [no subsidy ⇒ no deficit]

another interpretation of the payment:

sum value of others (in absence of i − in presence of i)

interpretation of the utility under VCG mechanism

$$v_i(f^{eff}(\theta_i, \underline{\theta}_{-i}), \theta_i) - p_i^{VCG}(\theta_i, \underline{\theta}_{-i})$$

$$= \underbrace{\sum_{j \in N} v_j(f^{eff}(\theta_i, \underline{\theta}_{-i}), \theta_j)}_{\text{max social welfare in presence of } i} - \underbrace{\max_{a \in A} \sum_{j \neq i} v_j(a, \theta_j)}_{\text{max social welfare in absence of } i}$$

= marginal contribution of i in the social welfare

Examples:

1. Single object allocation. Type = value for the object

if allocated, the agent gets this value and zero otherwise.

$$p_i^{VCG}(\theta_i, \underline{\theta}_i) = \max_{a \in A} \sum_{j \neq i} v_j(a, \theta_j) - \sum_{j \neq i} v_j(f^{eff}(\theta_i, \underline{\theta}_i), \theta_j)$$

efficient allocation would give the object to the individual whose reported type is highest.

Consider 4 players, types: $\{10, 8, 9, 5\} \Rightarrow \{9, 0, 0, 0\}$

(2) What is pivotal in the VCG payment?

3 players having the following valuations

	Football	Library	Museum
A	0	70	50
B	95	10	50
C	10	50	50

VCG allocation : M (maximizes SW)

A pays = 105 − 100 = 5

B pays = 120 − 100 = 20

C pays = 100 − 100 = 0 ← non pivotal agent

The agent whose presence changes the outcome is charged money

They are the pivotal players.

③ Combinatorial allocation : sale of multiple objects

	ϕ	$\{1\}$	$\{2\}$	$\{1,2\}$
θ_1	0	8	6	12
θ_2	0	9	4	14

value is the type itself

$v_i(a, \theta_i) = \theta_i(a)$

Efficient allocation : $\{1\} \to 2$ and $\{2\} \to 1$: call this a^*

$$p_1^{VCG}(\theta_1, \theta_2) = \max_{a \in A} \sum_{j \neq 1} \theta_j(a) - \sum_{j \neq 1} \theta_j(a^*)$$

$$= 14 - 9 = 5 \quad ; \text{ payoff} = 6 - 5 = 1$$

$$p_2^{VCG}(\theta_1, \theta_2) = 12 - 6 = 6 \quad ; \text{ payoff} = 9 - 6 = 3$$