Addendum to transform Bayesian game to complete information NFG.

$$\bar{N} = \bigcup_{i \in N} \Theta_i = \{\theta_1^1, \theta_1^2, \dots, \theta_1^{|\Theta_1|}, \dots \}$$

new player set

Consider two players, type sets $\Theta_1 = \{\theta_1^1, \theta_1^2\}$, $\Theta_2 = \{\theta_2^1, \theta_2^2\}$

utility of player θ_1^1 — original payoffs of Bayesian game

$$\bar{u}_{\theta_1^1}(a_{\theta_1^1}, a_{\theta_1^2}, a_{\theta_2^1}, a_{\theta_2^2}) = P(\theta_2^1 | \theta_1^1)\, u_1(a_{\theta_1^1}, a_{\theta_2^1}, \theta_1^1, \theta_2^1) + P(\theta_2^2 | \theta_1^1)\, u_1(a_{\theta_1^1}, a_{\theta_2^2}, \theta_1^1, \theta_2^2)$$

[defining $a_{\theta_1^1} = a_1(\theta_1^1)$, $a_{\theta_1^2} = a_1(\theta_1^2)$ etc.] --- (1)

consider a mixed strategy $(\sigma_{\theta_1^1}, \sigma_{\theta_1^2}, \sigma_{\theta_2^1}, \sigma_{\theta_2^2})$ in this new game

~~mixed~~

$$\bar{u}_{\theta_1^1}(\sigma_{\theta_1^1}, \sigma_{\theta_1^2}, \sigma_{\theta_2^1}, \sigma_{\theta_2^2}) =$$

$$\sum_{a_{\theta_2^2} \in A_2} \sum_{a_{\theta_2^1} \in A_2} \sum_{a_{\theta_1^2} \in A_1} \sum_{a_{\theta_1^1} \in A_1} \sigma_{\theta_1^1}(a_{\theta_1^1})\, \sigma_{\theta_1^2}(a_{\theta_1^2})\, \sigma_{\theta_2^1}(a_{\theta_2^1})\, \sigma_{\theta_2^2}(a_{\theta_2^2}) \times \underbrace{\bar{u}_{\theta_1^1}(a_{\theta_1^1}, a_{\theta_1^2}, a_{\theta_2^1}, a_{\theta_2^2})}_{\text{now plug this in from (1), irrelevant } a_{\theta_i} \text{ terms will sum to 1}}$$

$$= \sum_{a_{\theta_2^1} \in A_2} P(\theta_2^1 | \theta_1^1)\, \underbrace{\sigma_{\theta_2^1}(a_{\theta_2^1})}_{=:\, \sigma_2(\theta_2^1, a_{\theta_2^1})}\, u_1(a_{\theta_1^1}, a_{\theta_2^1}, \theta_1^1, \theta_2^1)$$

$$+ \sum_{a_{\theta_2^2} \in A_2} P(\theta_2^2 | \theta_1^1)\, \underbrace{\sigma_{\theta_2^2}(a_{\theta_2^2})}_{\cancel{\text{this is}}\ \text{define as}\ \sigma_2(\theta_2^2, a_{\theta_2^2})}\, u_1(a_{\theta_1^1}, a_{\theta_2^2}, \theta_1^1, \theta_2^2)$$

$$= \sum_{\theta_2 \in \Theta_2} P(\theta_2 | \theta_1^1)\, U_1(\sigma_1, \sigma_2 | \theta_1^1)$$

$(\sigma_{\theta_1^1}, \sigma_{\theta_1^2}, \sigma_{\theta_2^1}, \sigma_{\theta_2^2})$

Hence a mixed strategy in the complete information game is a mixed strategy (σ_1, σ_2) in the Bayesian game.

It follows that the MSNE in that game will be a BE in this game.