भारतीय प्रौद्योगिकी संस्थान मुंबई

Indian Institute of Technology Bombay

CS 6001: Game Theory and Algorithmic Mechanism Design

Week 10

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Slide preparation acknowledgments: Onkar Borade

ज्ञानम् परमम् ध्येयम्

Knowledge is the supreme goal

Contents

The Vickrey-Clarke-Groves Mechanism (VCG)

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- Interpretation of the **payment**: **sum value of others (in absence of i − in presence of i)**
- Interpretation of the **utility** under VCG mechanism:

$$v_i(f^{AE}(\theta_i, \theta_{-i}), \theta_i) - p_i^{VCG}(\theta_i, \theta_{-i}) = \sum_{j \in N} v_j(f^{AE}(\theta_i, \theta_{-i}), \theta_j) - \max_{a \in A} \sum_{j \neq i} v_j(a, \theta_j)$$

$$= \text{marginal contribution of } i \text{ in the social welfare}$$

An Observation on VCG Mechanism

Utility under VCG mechanism:

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- Intuition:
  - **choice set monotonicity** says that with more agents, set of alternatives never reduces
  - **no negative externality** says if an agent is removed, that agent does not have a negative value towards the AE allocation without that agent

Examples

1. **Single Object Allocation**
   Type = value of the object if allocated, the agent get this value and zero otherwise

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   - This is **second price auction**

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   3 players having the following valuations :

	Football	Library	Museum
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- Payments
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  C pays $= 100 - 100 = 0 \leftarrow$ **non-pivotal agent**
- The agent whose presence *changes the outcome* is charged money
  They are the **pivotal** players

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3. Combinatorial Allocation: sale of multiple objects
3 players having the following valuations (value is the type itself $v_i(a, \theta_i) = \theta_i(a)$)

	$\emptyset$	$\{1\}$	$\{2\}$	$\{1\&2\}$
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- $p_2^{VCG}(\theta_1, \theta_2) = 12 - 6 = 6$, payoff $= 9 - 6 = 3$

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- Also assume **selfish valuations**, i.e., $\theta_i(a) = \theta_i(a_i)$, agent i's valuation does *not* depend on the allocations of others

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Claim

In the allocation of goods, the VCG payment for an agent, that gets no object in this efficient allocation, is zero

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- $p_i^{VCG}(\theta) = \sum_{j \in N} \theta_j(a_{-i}^*) - \sum_{j \in N} \theta_j(a^*) \leqslant 0 \implies p_i^{VCG}(\theta) = 0$

VCG Mechanism in Combinatorial Allocations

Definition (Individual Rationality)

A mechanism (f, p) is *individually rational* if $v_i(f(\theta), \theta_i) - p_i(\theta) \geqslant 0, \forall \theta \in \Theta, \forall i \in N$

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\begin{aligned}
\text{utility of player } i = \theta_i(a^*) - p_i^{VCG}(\theta) &= \theta_i(a^*) - \Big(\sum_{j \neq i} \theta_j(a^*_{-i}) - \sum_{j \neq i} \theta_j(a^*)\Big) \\
&= \sum_{j \in N} \theta_j(a^*) - \sum_{j \neq i} \theta_j(a^*_{-i}) - \theta_i(a^*_{-i}) + \theta_i(a^*_{-i}) \\
&= \underbrace{\sum_{j \in N} \theta_j(a^*) - \sum_{j \in N} \theta_j(a^*_{-i})}_{\geqslant 0, \text{ by definition of } a^*} + \underbrace{\theta_i(a^*_{-i})}_{\geqslant 0} \geqslant 0
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   — Real-time bidding, automated bidding, decisions on the fly possible

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- **Note:** actual implementation in practice might be different, here we discuss only the principle of its computation

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- This is the total expected payment, to convert this to the pay-per-click: $\frac{1}{p_i \hat{\rho}_i} p_i^{VCG}(b)$

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   - If 1 and 2 collude and bid higher, both of them reduce their payments $\implies$ utility increases

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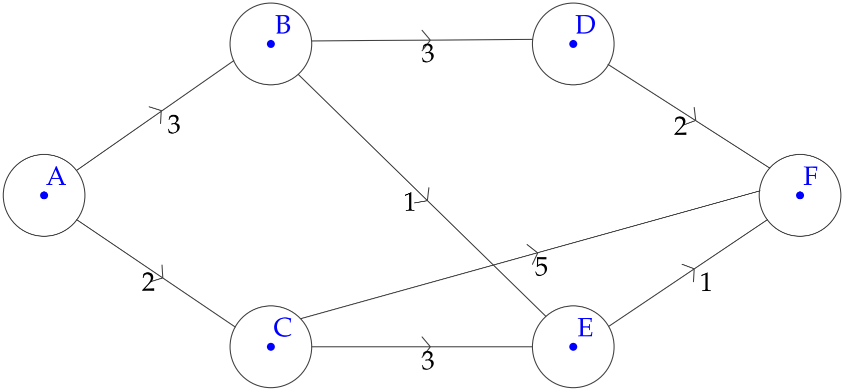

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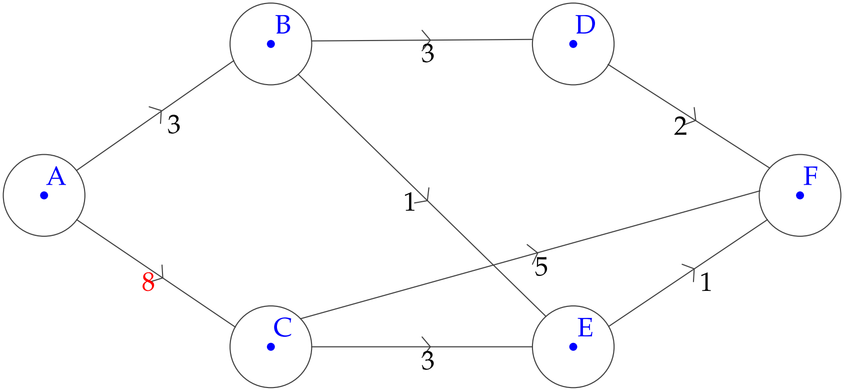

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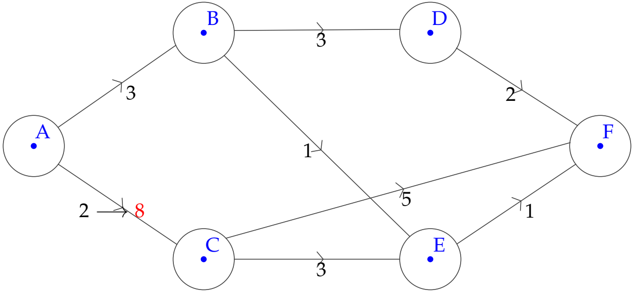

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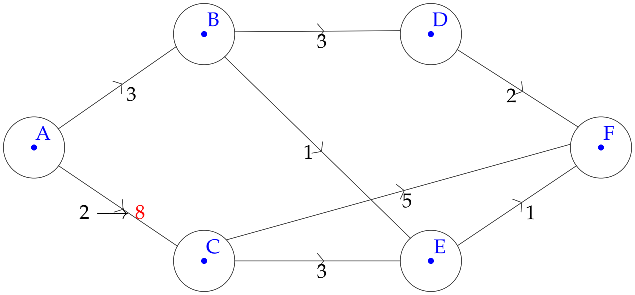

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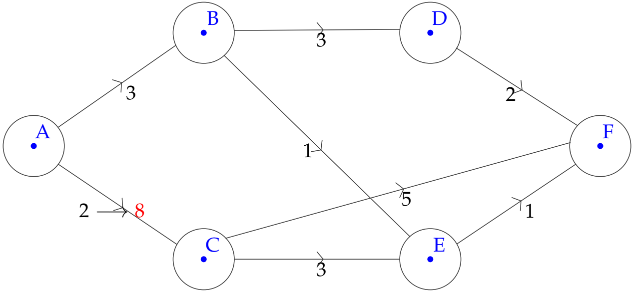

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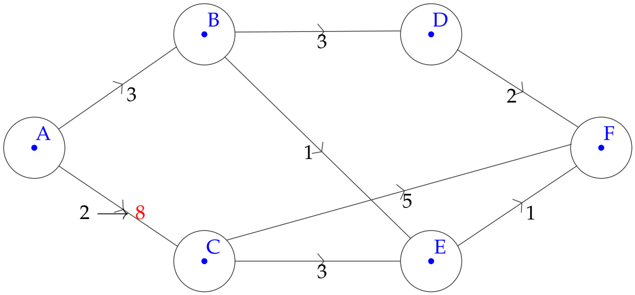

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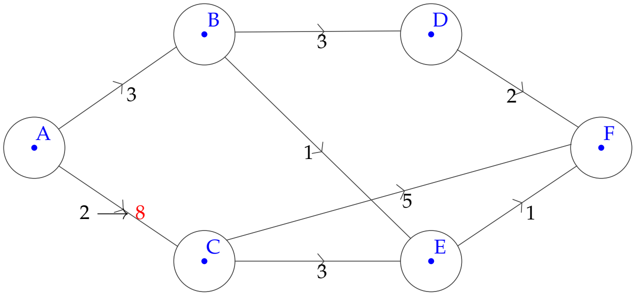

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   - If the players are partitioned into two groups and the surplus of one group is redistributed over the other group, then it is budget balanced, but the overall efficiency is compromised

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These are certain limitations that are good to know for effective use of VCG, however, it is the most widely used mechanism in the literature

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