Matching returned by TTC is unique. (simple)

Suppose it is not unique, $\exists$ a' different from a^{TTC} which is also a core matching. Suppose a' and a^{TTC} differ only in round k. That means the allocations from R1 to R(k-1) of TTC are identical and in k^{th} round $a'(i) \neq a^{TTC}(i)$ for some i. But since TTC allocates the most preferred houses available in R(k) to the agents that got allocated in Round k, this group can deviate and do the allocation like TTC that gives at least one agent strict better house and everyone else the same. Hence, $\nexists$ any such other core matching a'.

Application of One-Sided Matching

Organ exchanges : more specifically, Kidney exchanges

The real world problem: many people suffer from organ diseases, e.g., liver, pancreas, kidney. (~~part of~~) In order to save them, medical sciences allow transplanting them from other living or deceased humans.

Focus on kidneys: hundreds of thousand people suffers from kidney ailments (in India, numbers are larger, the data we have is from US) – unos.org to learn more.

DGHS NOTTO – national organ & tissue transplant org. notto.nic.in

☆ One option for them is to go for regular dialysis

Other option is transplant

The second one has a better quality of life and overall less costly.

The donated organ may come from a deceased person or a living donor, e.g., close relatives, etc.

But transplant needs more

– The donor and the patient need to be medically compatible

Family member/relative —/→ patient (not compatible, willing)

Blood-type compatibility

Tissue type compatibility

more points to consider

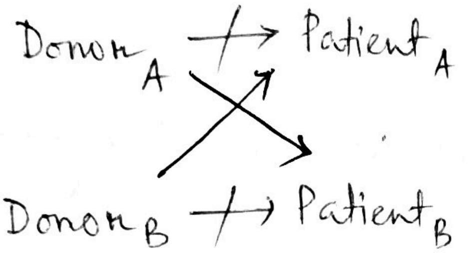
First successful kidney transplant was by Dr. Joseph Murray in 1954, on two monozygotic twins, it survived for 8 years. Got Nobel prize in medicine in 1990.

In 1971, India's first successful kidney transplant happened in CMC, Vellore, by Dr. Mohan Rao.

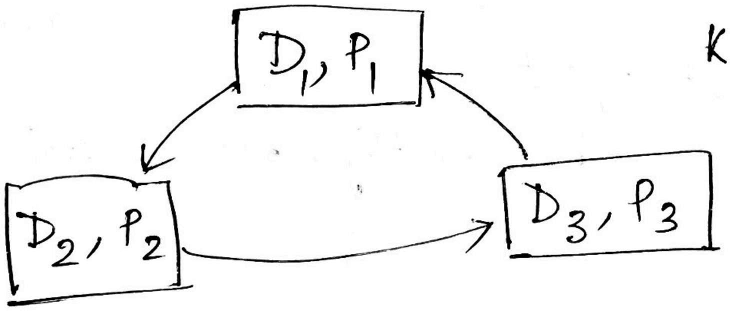
As of 2021, CMC alone has conducted 3755 transplants.

Initially, all transplants were ad-hoc, perhaps via chance. Sometimes even illegally in exchange of money.

Something more is possible if the donor is incompatible to the patient



Hence an exchange of the donors is possible



Kidney ≡ House

preferences ≡ medical compatibility

Cycles can be longer as well.

The first kidney paired exchange happened in 1986.

The first ~~re~~ medically recorded KPD in 1991, South Korea.

How to match the KPs for a donation? (VIA TTC)

Easiest is to use the standard framework of house allocation. Several desirable properties, but some limitations too.

Preferences in house allocation is organic

in kidney exchange is medically determined

3-3

Advantages

1. The TTC is in the core — no group of KPs can deviate and get a better allocation.
2. Every patient is weakly better off by participating (individual rationality — haven't proved formally, but holds for TTC)
3. Poly-time

Disadvantages

1. In practice, there can be patients without a donor and deceased donors/altruistic donors

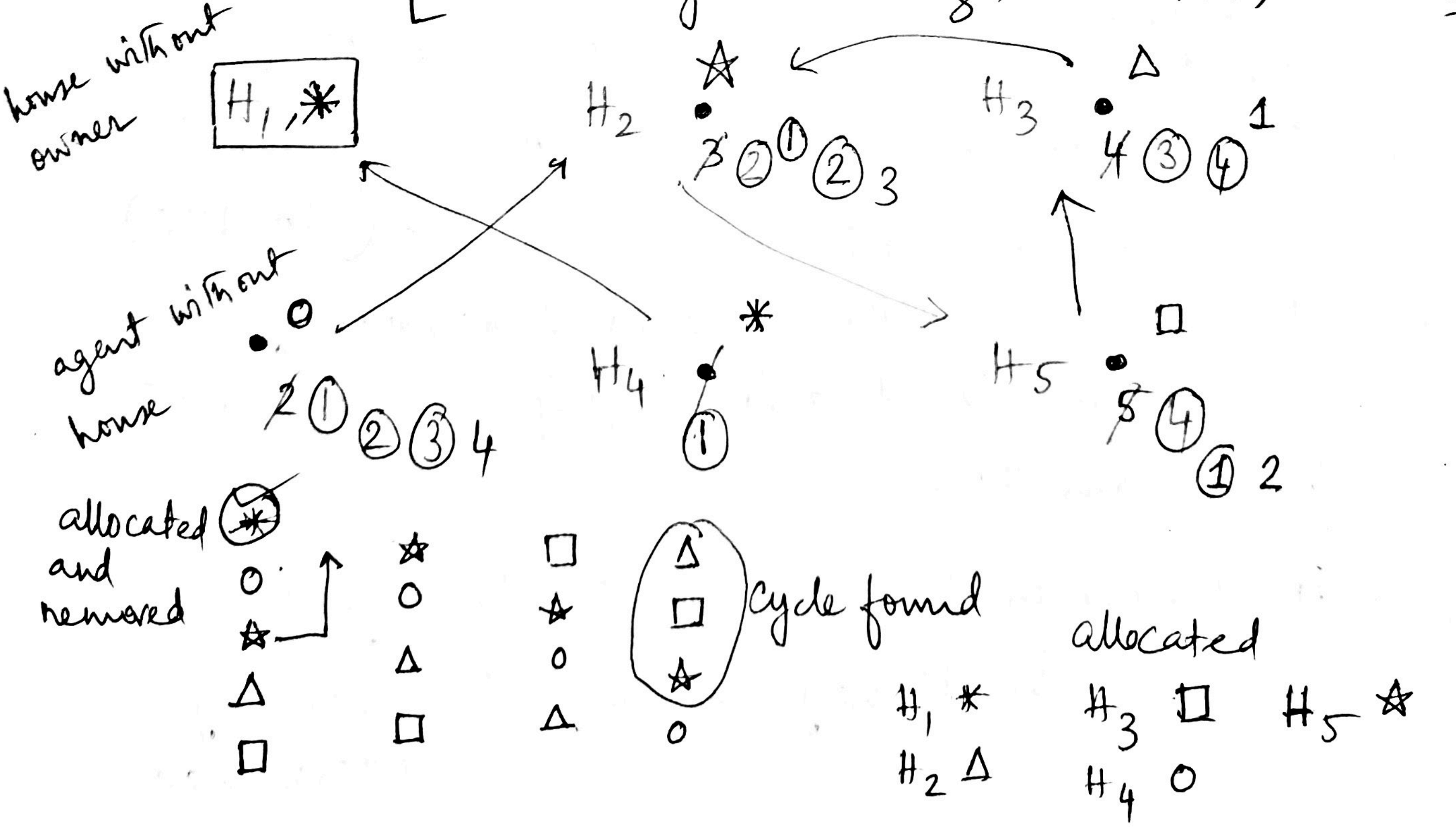
   deceased donors/altruistic donors → house without agent

   patients without a donor → agent without house

   TTC doesn't work directly, but a variant works

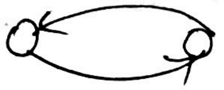
   "you request my house — I get your turn"

   [Abdulkadiroglu & Sonmez, JET 1999, ~~GEB 1999~~]



The mechanism satisfies PE, strategyproofness and several other properties. See Sonmez, Unver (2009) GEB paper for details.

2. Long cycles are difficult to execute medically

 requires 4 surgeries

a cycles of size $|S|$ nodes require $2|S|$ surgeries

Incentive issue: The surgeries need to be simultaneous.

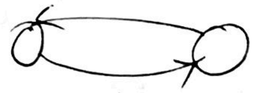
- no contracts are possible with organs
- if the surgeries are done on different times/days, the donor of the patient who received an organ can back-off. To avoid this simultaneity is necessary,

3. Strict preference ordering is too much than necessary.

In practice, only compatible/incompatible is fine.

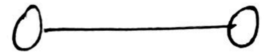
(Binary preferences) all compatibles in one equivalence class.

Proposal two: <u>via matching</u> (graph theory)

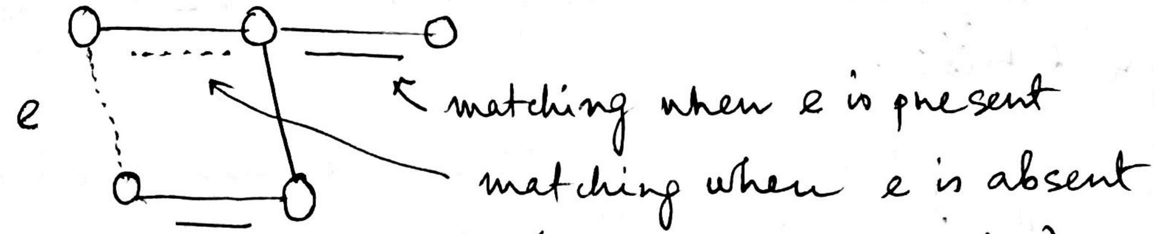
Consider only two cycles 

larger cycles are logistically impossible in practice.

Two cycle is represented with undirected edge.



Kidney exchange using Matchings (Graph)



Given an undirected graph (of patient-donor pairs)

find a maximum cardinality matching

↑ can lead to strategic manipulations.

- Can also be easily handled

Algorithm (Priority Matching) [Roth, Sonmez, Unver, JET 2005]

Consider a priority order $1, 2, \ldots, n$ over the vertices

M_0 = set of maximum cardinality matchings over the given graph

For $i = 1, 2, \ldots, n$:

Let S_i be the set of matchings in M_{i-1} that matches vertex i

If $S_i \neq \phi$:

$M_i \leftarrow S_i$

else

$M_i \leftarrow M_{i-1}$

Return M_n

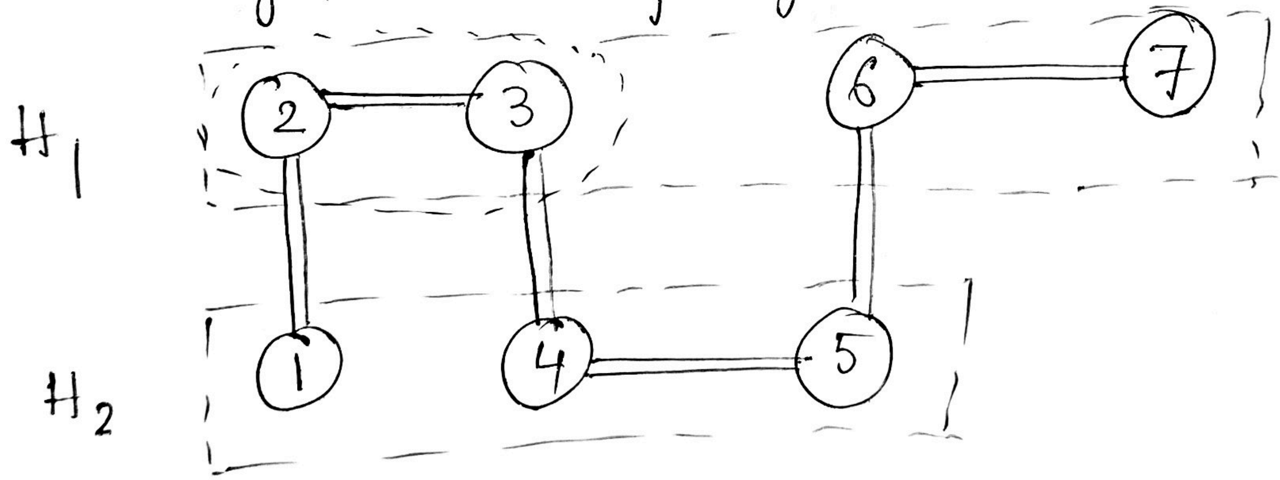
The priority order also has medical significance.

Advantages:

1. Each matching in M_n matches the same set of vertices
2. No agent underreports the set of its compatible edges
3. Pareto optimal. No other matching can match a superset of the matched agents.
4. Polynomial time. ~~Uses~~ Uses Edmonds-Gallai decomposition from graph theory.

Disadvantages:

Kidney exchanges currently are done via the hospitals. Hospitals are the new players - since hospitals share the data to a ~~cen~~ centralized exchange for running the matching algorithm

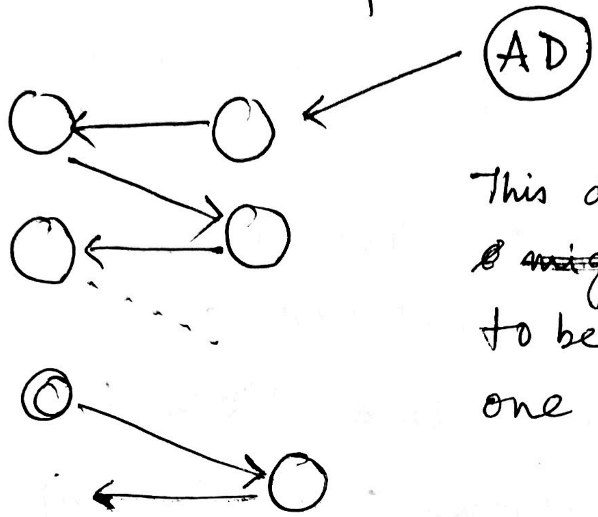


under the matching across hospitals, H_1 may not get one of its patient matched. It can stop reporting the data of 2 and 3 and be better off.

Maximum matching may not be the ~~most~~ safest thing.

3-7 Altruistic donor (post 2007)

Donors without a patient. "Good Samaritan" donor



This donation chain ~~might~~ is not required to be finished in one go / simultaneously.

Alvin Roth, Lloyd Shapley got Nobel Prize in Economics 2012
"for the theory of stable allocations and the practice of market design".