Algorithmic Mechanism Design for Egalitarian and Congestion-aware Airport Slot Allocation

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Abstract

We propose a game-theoretic model and a mechanism design solution to allocate slots fairly at congested airports. This mechanism: (a) ensures that the slots are allocated according to the *true* valuations of airlines, (b) provides fair opportunities for flights connecting remote cities to large airports, and (c) controls the number of flights in each slot to minimize congestion. Drawing inspiration from economic theory, this mechanism allocates the slots based on an *affine maximizer* allocation rule and charges payments to the airlines to incentivize them to participate in the allocation process and reveal their actual valuations. The allocation also optimizes the occupancy of each slot to keep them as uncongested as possible. The formulation solves an optimal integral solution in strongly polynomial time. We conduct experiments on the data collected from two primary airports in India. We also compare our results with existing allocations and an allocation based on the International Air Transport Association (IATA) guidelines. The computational results show that our mechanism is more egalitarian and generates 20%-30% higher *social utility* than the IATA based state-of-the-art approach and current allocations.

Keywords: Airport slot allocation, Congestion mitigation, Remote city connectivity, Strategic behavior, Game theory, Mechanism design, Strongly polynomial algorithm.

1 Introduction

In the last two decades, an overwhelming increase in demand for air transportation coupled with political, physical, and institutional constraints for capacity expansion resulted in congestion and delays at the world's major commercial airports. As the demand for airport slots[1] exceeds its available capacity; it results in more holding time for the permission to land or take-off, leading to congestion. Airport congestion veritably imposes a tremendous cost on the global economy, including additional aircraft operating costs, and passenger delay costs. The other externalities are environmental and noise pollution around the congested airport, while aircraft wait in queue with engines fired up. According to the study commissioned by the Federal Aviation Administration, the total delay cost[2] in the United States for the calendar year 2019

[1]The daily runway scheduling period of an airport is divided into time intervals of a fixed length (e.g., 15 minutes) called slots (Androutsopoulos and Madas, 2019). We define 'slot' as a permitted time-interval for a planned operation to use the full range of airport infrastructure necessary for an aircraft to arrive or depart. The slot allocation to an airline movement means the permission to use the airport for landing or taking-off within a particular time interval (slot) (Pellegrini et al., 2017).

[2]The total cost of flight delays is the sum of the costs of delay to airlines, passengers, lost demand, and indirect costs to business sectors which depend on air transportation of their goods and services.

is estimated at $33 billions (FAA, 2020). The situation is similar in other parts of the world, which is predicted to worsen in the future.

However, governments worldwide have used various administrative instruments to promote remote air connectivity. Air transportation has emerged as a 'lifeline service' to connect remote cities where land transportation is not an option (Fageda et al., 2018). The link between air connectivity and economic growth is well established (Brueckner, 2003; Alderighi et al., 2017). Bilotkach et al. (2015) show a strong correlation between regional development and connectivity to various new destinations. Fageda et al. (2018) observe that an appropriate mechanism can help governments effectively promote regional air transport, which otherwise is excluded under normal market conditions. An administrative approach might respond to a country's varied political goals, for example, United Kingdom (UK) governments' objective to connect Heathrow airport to the rest of the UK (Burghouwt, 2017). However, in market-based mechanisms for slot allocation, e.g., auction; the airlines are more inclined toward servicing between metro cities. Airlines try to suppress frequencies in less profitable routes and allocate the resources to the most profitable ones (Leandro et al., 2021). The low profitability and inconsistent load factor of movements from remote cities result in low valuations for those flights. This study addresses congestion awareness and promotes remote connectivity during airport slot allocation.

The runway resources at congested airports in peak periods are valuable and scarce. Administrative approaches such as grandfather rights, first-come-first-serve, lotteries, and the new entrant rule are inefficient and random ways to allocate slots at congested airport (Zografos and Madas, 2003; Zografos et al., 2013). Therefore, the slots should be allocated to those airlines valuing them the most (Cao and Kanafani, 2000). The slot allocation by the International Air Transport Association (IATA) follows the *use-it-or-lose-it* rule *(80/20 rule)*. By this rule, if an airline uses at least 80% of the slots allocated in a predefined period, the airline can continue using the same slots in the next period. Although this rule offers airlines the certainty to publish tickets in advance allowing consumers to plan travel, incidents occur when airlines use the slots with almost empty flights to maintain the record for at least 80% not to lose the slot. The lost slots are traded again and, increases the risk of seeing rivals take over the slot. These empty flights are often called *ghost flights*. One such incident caught attention British Mediterranean Airways flew empty six days a week for almost six months (when the service from London to Tashkent was abruptly canceled) until they started another route using the slot[3]. Though such incidents are not frequent, the mechanism should ensure that the scarce resources are allocated to those agents[4] who truly value them.

With maintaining the social goals to mitigate congestion and *egalitarian* allocation of slots (the process of slot allocation should create equal opportunity to the currently disadvantaged remote cities by incentivizing the airlines to increase connectivity with them), we aim to assign the slots to the airlines (or flights) who value them the most. It might not always be possible to achieve multiple objectives simultaneously, e.g., finding an efficient and egalitarian allocation of resources is not always feasible. Therefore, we try to create a balance between these objectives.

Since airlines are independent entities, they might misreport their privately known values for the slots to secure their preferred slots, making the situation like a game where airlines strategically choose their actions. Suppose we aim at an efficient allocation of slots. In that case, a strong need to ensure *truthfulness* arises, meaning we must design a mechanism that incentivizes the airlines to tell the truth about their private information. We should consider the liberty owned by the airlines to choose whether to participate in the implemented mechanism. Moreover, suppose we want to achieve a specific objective requiring each airlines' collaboration. In that case, the mechanism must ensure *individual rationality*, meaning that airlines rationally

[3] http://news.bbc.co.uk/2/hi/uk_news/6441103.stm

[4] We use the terms agent, player, and movement interchangeably in this paper.

choose to participate in the mechanism. This property along with truthfulness will imply the successful implementation of the mechanism to achieve the desired objective.

Efficient and truthful mechanism exist but they allow utility transfers between the agents. In those mechanisms, the sum of the transfers between the agents might not always be zero; in other words, the mechanism might not be *budget-balanced*. An impossibility result by Green and Laffont (1979) proposes that no mechanism exists that simultaneously guarantees efficiency, strategy-proof, and budget-balance. In other words, the sum of airlines' payments to obtain the slots might be more than zero (meaning the planner earns in that mechanism) or less than zero (meaning the planner needs an outside subsidy to operate that mechanism). If the sum of payments exceeds zero, the planner's collected revenue can be used in the airport's facilities. However, it is always undesirable for the planner to be required to pay for allocating the slots; in other words, the planner needs an outside subsidy to operate. Therefore, we need a weaker notion of budget-balance known as *weakly budget-balance*, meaning that the planner might gain some money but never runs into a deficit while allocating the slot.

For a large airport, the number of movements and slots and capacities of slots is insignificant, exponentially increasing in the size of the set of all feasible solutions. Finding the optimal allocations for each flight can become computationally expensive. Therefore, we desire to design a mechanism whose outcome can be computed with minimum resources for computation, e.g., time and storage. Next, we briefly introduce the work done in this paper by summarizing the key contributions.

1.1 Our contributions

This study provides a game-theoretic model for the multiagent interaction of airlines. We consider a central planner who allocates the slots among the airlines, maintaining the feasibility constraints and concerning some social objectives.

We devise a slot allocation mechanism (egalitarian and congestion-aware truthful slot (`ECATS`)), to integrate various (administrative and market-based) instruments into an overall slot allocation strategy that finds a balance between multiple objectives. These objectives are (a) efficiency (slots should be given to the agents who value them the most), (b) egalitarian (slot allocation should create equal opportunity to remote cities by incentivizing the airlines to increase connectivity with the remote cities), and (c) the allocation should mitigate the congestion at the airport.

We avoid rational agents' manipulation by including (d) truthfulness and (e) individual rationality. Our objectives include (f) weakly budget-balance to prevent the payments on the planner, and we desire that the mechanism be (g) computationally tractable, meaning it requires fewer computational resources to find the optimal allocation. Note that a dearth of models exists in the extant literature that can simultaneously consider all the above objectives in airport slot allocation. We summarize the main contributions of this paper as follows.

- We provide a game-theoretic model for the slot allocation problem.
- The proposed mechanism (`ECATS`) incentivizes airlines to report the actual valuations for the slots (Theorem 1).
- `ECATS` ensures that the airlines can never be worse off by participating in the allocation process; hence, it guarantees their voluntary participation (Theorem 2).
- `ECATS` ensures that the planner never needs an outside subsidy to operate (Theorem 3).
- The slot allocation problem is an integer program belonging to a computationally intractable class (NP-complete). However, our formulation is solvable in strongly polynomial time

(Theorem 6).

- We propose the remote city opportunity factor (Equation (3)) that works as tunable parameters to ensure a more egalitarian slot allocation that increases the connectivity with remote cities as shown in the experiments (Figures 3 and 4).
- We collected the flight movement data for Delhi and Chennai airports. We evaluate `ECATS` by comparing its allocation with the current allocation and a state-of-the-art allocation given by Ribeiro et al. (2018) based on IATA guidelines. The experiments show that the social utility generated using `ECATS` is $5\% - 20\%$ and $20\% - 30\%$ higher than the IATA-based state-of-the-art and current allocations, respectively (Figures 5 and 6).

2 Literature review

Contemporary research on airport slot allocations primarily focuses on demand-side solutions to mitigate congestion because can restore the demand-capacity balance over a medium to short time horizon with low investments (Barnhart and Cohn, 2004). The demand management strategies to manage slots at congested airports range from various administrative tools to market-based mechanisms.

Administrative approaches: The administrative approaches for slot allocation include grandfather rights, first-come-first-serve, lotteries, and the new entrant rule (Ball et al., 2006). These approaches are government or institutional interventions through rules and regulations to allocate scarce slots at congested airports.

- The historical precedence of the allocated slots is maintained in the grandfathering rights (Sieg, 2010). Airlines can exercise their historical usage rights during the slot allocation step. The grandfather rights implement a use-it-or-lose-it rule, where an airline must use the allocated slot at least 80% of the time to obtain the slot in the next allocation period. Based on the historical precedence, Ribeiro et al. (2018) proposed a priority-based slot allocation model (PSAM) that minimizes the displacement of airlines' slot requests. The model fully complies with the 'primary criteria' of the IATA guidelines and the historical precedence.
- In the first-come-first-serve rule, the slots are assigned based on arrival time, and airlines queue up for runway and gate access. The first-come first-serve concept was used at the five most congested airports, three airports in New York (JFK, LaGuardia, and Newark), Chicago O'Hare and Washington Reagan, to manage congestion and delays (Fan and Odoni, 2002).
- In the new entrant rule, preference is given to new entrants for the vacant/newly available slots. For example, numerous slots at the newly constructed runway at Frankfurt airport were allocated to new entrants (IATA, 2017). In India, the allocation is governed by the Airports Authority of India (AAI), mandating that 50% of the slots contained in the pool at initial slot allocation must be allocated to new entrants.

Administrative instruments might respond to a country's varied social or political goals, For example, the UK government's objective is to connect Heathrow airport to the rest of the UK (Burghouwt, 2017).

Limitations of administrative approaches: The extant literature highlights several issues with administrative approaches, with one of the biggest criticisms being that they are economically inefficient. Another criticism of the grandfathering rule is that it creates entry barriers to new airlines (Vaze and Barnhart, 2012) and encourages legacy carriers to overschedule flights to avoid losing the allocated slots (Harsha, 2009). Furthermore, it prevents adequate competitive pressure on these incumbents from the new entrants, especially the low-cost carriers. For example, British Airways holds 51% of the take-off and landing capacity at London Heathrow (the busiest airport in Europe), estimated at 742 million pounds (Horton, 2020). This airport operates at 100% capacity, making it incredibly challenging for new entrants to obtain a slot. Moreover, when no monetary payments are involved in slot allocation, the airlines might over demand the slots to minimize deviation from their preferred schedule (Vaze and Barnhart, 2012) or gain market power through slot hoarding (Sheng et al., 2019). Historically, government interventions lead to inefficiencies, and airlines might exploit loopholes in the regulations (Burghouwt, 2017).

Market-based mechanisms: Researchers have shown that market-based mechanisms result in the efficient allocation of scarce resources (Zhang et al., 2020; Abdulkadiroğlu and Sönmez, 2003). Czerny and Zhang (2014) concluded that the market mechanisms would reveal the resources' true economic value at the congested airport. They provide a flexible and transparent approach for balancing the supply-demand mismatch through pricing. We find applications of such mechanisms in ride-sharing (Li et al., 2022), allocating charging stations (Rigas et al., 2020) to public car parking resource problems (Wang et al., 2020). These mechanisms efficiently allocate scarce resources to those who value them the most. In the case of airport slot allocation, market based mechanisms provide equal opportunities to the legacy carriers and new entrants and promote healthy competition. Various market-based mechanisms have been proposed, such as congestion pricing, auctions, and secondary slot trading. Congestion pricing favors charging airlines a fee based on several proposed rules such as:

- Pricing based on the marginal cost of delays (Vickrey, 1969; Carlin and Park, 1970)
- Differential pricing across carriers by considering small and large airlines (Brueckner, 2009)
- Pricing based on contributions to the infeasibility of the ideal solution (Castelli et al., 2012)
- Pricing based on passengers types when airlines price discriminates (Czerny and Zhang, 2014)
- Pricing based on the desired amount of delays that airlines are ready to buy (Mehta and Vazirani, 2020)

Congestion pricing results in considerable welfare gains, which can minimize total delay and attain ideal slot allocations (Daniel and Harback, 2009; Czerny, 2010). Daniel (2011) reported that congestion pricing could help in saving \$72 million to \$105 million annually at Canadian airports by reducing delays and associated costs. A drawback of congestion pricing is that the fee must be iteratively varied in time and among different airports depending on the degree of congestion set up by the airport administration. The other market-based approach widely discussed in the extant literature is auctioning, where airlines bid for slots (Sheng et al., 2015). An auction-based mechanism maximizes the airport's revenue because it allocates slots purely based on valuation maximization (Harsha, 2009). Basso and Zhang (2010) compared congestion pricing with slot auctions and concluded that there is no clear winner for airport profit maximization.

Ball et al. (2020) proposed a quantity-contingent-based auction mechanism to allocate the slots at congested airports. The authors proposed using Vickrey Clarke Groves (VCG) mechanism and imposed a constraint on the accepted bids to control airline's market power (Vickrey, 1961; Clarke, 1971; Groves, 1973); however, they did not consider issues like individual rationality and computational complexity. Pertuiset and Santos (2014) used the VCG mechanism to allocate slots in congested European airports. The properties of the model are similar to VCG as it ensures efficiency and incentive compatibility. The authors recommended auctioning 10%

of slots annually, which would eventually phase out and disappear in the historical slots within a decade. Rosenthal and Eisenstein (2016) used the VCG mechanism to address rescheduling delayed flight arrivals by constructing a payoff matrix of moving the assigned arrival slots either earlier or later. The related literature from the auction theory (in other applications) relates to this work. However, those approach either lead to computational tractability issues (Sandholm et al., 2005), or low revenue generation, even when the items sold are valuable (Ausubel et al., 2006). In particular, the VCG mechanism is suitable only when the goal is to achieve efficiency truthfully (to maximize the sum of participant's valuations).

This paper has considered multiple objectives relevant to airport slot allocation, e.g., congestion-awareness, egalitarianism, strategic guarantees (truthfulness and individual rationality) and computational tractability. Note that, achieving efficiency can be nonegalitarian, i.e., it might yield a biased allocation to specific movements because it aims to maximize the sum of the valuations. Our study on airport slot allocation differs from the VCG or mechanisms for parking allocation (Wang et al., 2020) in the following ways:

1. We consider the objective to connect remote cities, for which we use a remote city opportunity factor (Equation (3)) that works as tunable parameters to ensure increases in remote city connectivity.

2. Our optimization is congestion-aware and aims to reduce it.

3. We show that these social and efficiency goals can be achieved computationally tractable.

Limitations of market-based approaches: Despite being regarded as the most efficient way to allocate slots at congested airports, market-based mechanisms have shortcomings. They are considered detrimental for flights from remote cities due to their low valuations compared to flights from big cities. The low profitability and inconsistent load factor of movements from remote cities might limit their ability to win slots at an auction or pay pure market-based congestion prices. Any mechanism solely based on the money transfers will be unfavorable to remote communities and exclude of air-service to these cities (Green, 2007; Harsha, 2009; Sheard, 2014).

Administrative instruments might be warranted to achieve social objectives by ensuring connectivity to peripheral regions and supporting the population of remote regions. Combining market-based and administrative instruments leads to a hybrid mechanism, which allocates the slots based on efficiency (valuation maximization) and ensures slot opportunities for flights connecting to remote cities. This study addresses this dual goal of valuation-based slot allocation with remote city connectivity (social objective) at congested airports.

The critical lever for congestion minimization is to limit the number of allocated movements (landings/take-offs) in a specific slot. The most important decision here is determining the number of movements to assign based on the trade-off between the cost of delay and resource use. Our proposed mechanism determines the number of movements in each slot based on the trade-off between an increase in valuations due to additionally allocated movements and the resultant increase in congestion cost. It considers a proper balance between flight delays and the extent of the services offered (the number of flights scheduled). As the number of movements scheduled in each slot decreases, the congestion level, and the resultant delays would decrease. Based on a case study of LaGuardia Airport, Li et al. (2010) found that airport congestion can be minimized by limiting the number of allocated movements in a slot. Similarly, Swaroop et al. (2012) found that more than two-thirds of the total system delays can be reduced by capping the slot allocation.

We propose that if the number of movements in a slot is allocated up to a specific limit, it might not result in significant congestion and delays. However, allocating movements in a slot beyond a specific limit will add congestion and delays. Therefore, we have imposed a penalty in congestion costs for each additional allocated movement beyond a limit. Determining the allocation limit depends on factors such as weather conditions in a particular season, aircraft mix and the skill of air traffic controllers and pilots. Historical airport data can be used to determine these allocation limits.

To our knowledge, `ECATS` is the first mechanism that simultaneously considers multiple aspects for slot allocation. The remainder of the paper is organized as follows: Section 3 formalizes the model and the desirable properties and Section 4 and Section 5 present the mechanism and theoretical properties. We present the experimental results in Section 6 and conclude the paper in Section 7.

3 Model descriptions and assumptions

We divide the availability of the airport in disjoint time intervals or *slots*[5]. Let the set $S = \{1, 2, \ldots, n\}$ be the set of time slots. Each slot $j \in S$ has a capacity C_j, which is the maximum number of flights or movements that can be accommodated simultaneously, $C_j \in \mathbb{Z}_+, \forall j \in S$. Let $M = \{1, 2, \ldots, m\}$ be the set of movements. Every movement has a potentially different valuation for the different slots in S. The valuation of movement $i \in M$ for slot $j \in S$ is denoted by $v_{ij} \in \mathbb{R}_{\geqslant 0}$. For every movement $i \in M$, let v_i be the vector of i's valuations for every slot in S and $V = [v_{ij}], i \in M, j \in S$. We represent the set of all feasible allocations of the flights to the slots as $\mathcal{A}$. An allocation $A \in \mathcal{A}$ is $A = [x_{ij}], i \in M, j \in S$, where, $x_{ij} = 1$ if movement i is assigned slot j and is equal to 0 otherwise. We assume that each movement can be assigned to at most one slot.

We will use the shorthand $v_i(A)$ to denote the valuation of movement i in the allocation A, i.e., it will be equal to v_{ik}, if i is assigned slot k in A, and zero if i is unallocated in A. The valuation v_{ik} implicitly reflects the importance of using the slot k for the agent i. We assume that the valuation vector v_i is private information of the agent i, for all $i \in M$.

As a running example to understand our mathematical model better, we consider the typical airport slot allocation, which happens once every six months. The movements are identified by their unique flight numbers. We assume that a specific *day's* (e.g., say Wednesday) slot $j \in S$ is allocated to a specific movement for that day for these six months, where S denotes the set of slots of that day.[6] This implies that after allocation, the movement will be able to use slot j of that day of the week for these six months. The valuation v_{ij} of agent i for slot j represents the cumulative value that the slot gives her over these six months. If there are certain times of the year when there is a disruption in the regular weekly timeslots, e.g., due to popular travel times or disruption due to pandemics, etc., the current model can be easily extended to allocate slots during those weeks separately without harming any of the main results of this paper.

Movements, in our discussions, mean the individual flights. While multiple flights may be run by a specific airline, they are uniquely identified by their *flight numbers* or *callsigns*. Therefore, it is easy to identify movements that are unique and can be considered as the players in the slot allocation game.

If the preferences of the agents are their private information and the mechanism has no additional structures (e.g., transfer of individual payoff), the only truthful mechanisms are

[5] We find that the term 'windows' and 'slot' are used interchangeably in the literature. Following the definition of slot provided by Androutsopoulos et al. (2020), Mehta and Vazirani (2020) and Ribeiro et al. (2018), we refer to each disjoint time interval as a slot with a defined capacity.

[6] This 'day of the week' allocation is necessary to accommodate the cases where a given flight may not operate on all days of the week. For example, the movement `6E 5353`, operating on Monday, Wednesday, and Friday, will contest for a slot only on those days.

dictatorial mechanisms (Roberts, 1979, Thm 7.2). A dictatorial mechanism is to always select a pre-selected agent's favorite outcome. Therefore, the use of transfers in some form is inevitable to ensure the truthfulness of the agents.

The planner decides the allocation A. To ensure truthfulness, the additional structures of transfers is added. The planner computes and charges payments $p = [p_i \in \mathbb{R}]_{i \in M}$ to each of the movements. We assume that every agent wants a more valued slot to be assigned to her and also wants to pay less. Therefore, the net payoff of an agent is assumed to follow a standard *quasi-linear form* (Shoham and Leyton-Brown, 2008)

$$u_i((A,p), v_i) = v_i(A) - p_i. \tag{1}$$

The scheduler does not know the actual valuations of the agents. To achieve an objective based on the true valuations of the agents, it needs the agents to report their valuations, and then it decides the allocation and the payments. This leaves the chance for an agent to misreport her true valuation, e.g., an agent can report a higher valuation to obtain a prioritized slot. To distinguish, we represent the true valuation and the reported valuations as v_{ij} and v'_{ij}, respectively. To denote the true valuation profile represented as an $m \times n$ real matrix, we use the shorthand $v = (v_i)_{i \in M}$, and v' denotes the reported valuation profile. The notation v_{-i} denotes the valuation profile of the agents except i. Therefore, the decision problem of the planner is formally captured by the following function.

Definition 1 (Airline Scheduling Function (ASF)). *An* airline scheduling function (ASF) *is a mapping $f : \mathbb{R}^{m \times n} \to \mathcal{A} \times \mathbb{R}^m$ that maps the reported valuations to an allocation and payment for every agent. Hence, $f(v') = (A(v'), p(v'))$, where A and p are the allocation and payment functions respectively* [7].

We assume that congestion and delays start kicking in when the number of movements in a slot exceeds a pre-defined threshold. We propose a division of the slot capacity into two parts: congestion-free and congestion-prone. Suppose the number of allocated movements in a time slot j exceeds a defined threshold, $(1 - \lambda)C_j$. In that case, we say that the slot j is congested. The determination of the threshold, λ, depends on various factors such as airport infrastructure and local weather conditions. Historical data can be used to determine this allocation limit. Therefore, the following term captures the congestion level in the slot corresponding to the allocation A and is equal to the number of allocated flights in the congestion-prone capacity.

$$e_j(A) = \left(\sum_{i \in M} (x_{ij}) - C_j(1 - \lambda) \right)^+ \quad \forall j \in S \tag{2}$$

Our results hold for e_j as any linear function of x. The notations and their descriptions are briefly summarized in the Table 1.

3.1 Remote city opportunity factor

Inadequate air connectivity to metro cities is considered a significant obstacle for the local economic development of remote cities. It is shown that poor air connectivity services inhibit local employment growth by limiting the city's attractiveness for new businesses and reducing the viability of existing businesses (Brueckner, 2003). In the existing literature, empirical studies from European Union, the United States, and China have established a positive impact of metro airport connectivity on remote cities' regional growth (Brafman and Tennenholtz, 2003; Yao

[7] We overload the notation A and d to denote both functions and values of those functions, since their use will be clear from the context.

Table 1: Notations used in the paper.

Symbol	Description
S	Set of disjoint time slots, $\lvert S\rvert = n$
M	Set of movements, $\lvert M\rvert = m$
C_j	Capacity of slot $j \in S$
v_{ij}	Valuation of movement $i \in M$ for slot $j \in S$ (say, e.g., in Indian rupees)
V	$[v_{ij}],\ i \in M, j \in S$
A	Allocation matrix, $A = [x_{ij}], i \in M, j \in S$, where, $x_{ij} = 1$ if movement i is assigned slot j and is equal to 0 otherwise.
p	$p = [p_i \in \mathbb{R}]_{i\in M}$, where p_i is the payment charged to movement i
λ	Fraction of capacity that is considered to be congestion prone
$e_j(A)$	Number of allocated flights according to A in the congestion-prone capacity of slot j
g	Per-unit congestion cost (in the same unit as the valuations, e.g., Indian rupees)
ρ_i	Remote city opportunity factor of the city to which the movement i is connecting the concerned airport

and Yang, 2008). A study based on airports in Canada shows an increase of 1126 additional air-travel passengers can create one person-year of employment (Benell and Prentice, 1993). Yao and Yang (2008) showed that a 10% increase in population density could lead to a 1.7% increase in air passenger volume in China. Therefore, the remote cities' adequate population is also an important criterion to make air connectivity from remote cities economically viable. By considering the above two factors (economic progress and population), we propose the following remote city opportunity factor (RCOF):

$$\rho_i = \alpha_i \frac{\gamma_{\max} - \gamma_i + \delta}{\sum_{i\in M}\{\gamma_{\max} - \gamma_i\} + \delta} + (1 - \alpha_i)\frac{\omega_i - \omega_{\min} + \delta}{\sum_{i\in M}\{\omega_i - \omega_{\min}\} + \delta} \tag{3}$$

where γ is the social progress index (SPI)[8] and ω is the population of the city. $\gamma_{\max}$ takes the maximum value of SPI, and $\omega_{\min}$ is the minimum population among all the cities in consideration. Here α_i indicates the relative weight assigned to SPI and population to calculate ρ_i. A small factor δ with $\lim_{\delta\to 0}$, is added to avoid a division-by-zero error.The value of ρ_i ranges between 0 to 1.

The SPI is an innovative way to measure the development of a region. It is a widely used index measured by the thirty-five indicators related to basic human needs, foundations of well-being, and opportunities for the city's progress. The framework is closely coherent with all sustainable development goals (SDGs) parameters. This rigorous but straightforward framework makes it an invaluable proxy measure of SDG performance. It captures a wide range of measures involving social and environmental factors, thus proving to be a monitoring and guiding mechanism for national policy decisions and assisting businesses in planning corporate social responsibility activities. The SPI values for various countries and their cities are available at `https://www.socialprogress.org`. For India, the social progress performance data of 562 districts is collected and maintained by the Institute for Competitiveness, India. As can be seen from the above equation, the formulation is designed to prioritize the cities with a lower value of SPI and a high population to benefit from air connectivity.

[8] `https://www.socialprogress.org/`

3.2 Desirable properties

In this section, we formally define a few desirable properties that an ASF should satisfy.

Since the mechanism can only access the movements' reported values, for a truly efficient slot allocation, it is needed that the reported valuations must be the true values. The following property ensures that every movement is incentivized to reveal the values truthfully.

DEFINITION **2** (Dominant Strategy Truthfulness). *An ASF $f = (A(\cdot), p(\cdot))$ is* truthful in dominant strategies *if for every $v_i, v_i' \in \mathbb{R}^n_{\geqslant 0}, i \in M$*

$$v_i(A(v_i, v_{-i})) - p_i(v_i, v_{-i}) \geqslant v_i(A(v_i', v_{-i})) - p_i(v_i', v_{-i}).$$

The inequality above shows that if the true value of agent i is v_i, the allocation and payment resulting from reporting it 'truthfully' maximizes her payoff *irrespective of the reports of the other agents.*

Consider a mechanism that has a particular objective based on the strategies of each airline and ensures truthfulness. Is this property enough to ensure that we will achieve the objective after implementing the mechanism? No. We should also consider the liberty already owned by the airlines to choose whether or not to participate in the implemented mechanism. Furthermore, if we want to achieve a certain objective that requires each airlines' collaboration, the mechanism must ensure that the airlines rationally choose to participate in the mechanism.

The following property ensures that it is always weakly beneficial for every rational agent to participate in such a mechanism.

DEFINITION **3** (Individual Rationality). *An ASF $f = (A(\cdot), p(\cdot))$ is* individually rational *if for every v, and $i \in M$*

$$v_i(A(v_i, v_{-i})) - p_i(v_i, v_{-i}) \geqslant 0.$$

It is always desirable that a planner does not run into a deficit while implementing the mechanism. This property is known as *weak budget balance* and is formally defined as follows.

DEFINITION **4** (Weak Budget Balance). *An ASF $f = (A(\cdot), p(\cdot))$ is* weakly budget balanced *(WBB) if for every $v_i, v_i' \in \mathbb{R}^n_{\geqslant 0}$*

$$\sum_{i \in M} p_i(v_i, v_{-i}) \geqslant 0.$$

For a large airport, the number of movements and slots and capacities of slots is large; this leads to an exponential increase in the size of $\mathcal{A}$. For such a setting, computing the allocations and payments for each agent can become exponentially expensive. Therefore, we desire to design a mechanism whose outcome can be computed in time polynomial of input size. We consider mechanisms that are *strongly polynomial* as defined below.

DEFINITION **5** (Strongly Polynomial). *The algorithm runs in strongly polynomial time if (Grötschel et al., 1993)*

1. *the number of arithmetic operations (addition, subtraction, multiplication, division, and comparison) in the arithmetic model of computation is bounded by a polynomial in the number of integers in the input instance; and*
2. *the space used by the algorithm is bounded by a polynomial in the input size.*

While defining the above properties, we mention that we have a certain objective that we want to optimize, maintaining the feasibility constraints of the slot allocation. By ensuring these properties, we guarantee that the formulation of the optimization problem is not vulnerable to the rational behavior of the agents. Next, we represent our objective function, which we call *social utility*, and the feasibility constraints via an LP formulation. In the following subsection, we discuss the notion we call *social utility*.

3.3 Formulation of the objective function

We have multiple objectives: (a) efficiency (slots should be given to the agents who value them the most), (b) egalitarian (the process of slot allocation should create equal opportunity to the remote cities by incentivizing the airlines to increase connectivity with the remote cities as well), and (c) the allocation should mitigate the congestion at the airport. Achieving all three objectives simultaneously is not feasible, e.g., an efficient allocation may not always be egalitarian. Therefore, we try to create a balance between these three objectives.

An *efficient* allocation is to find the allocation A that maximises the sum of valuations of the movements, i.e.,

$$\operatorname*{argmax}_{A} \sum_{j \in S} \sum_{i \in M} v_{ij}\ x_{ij}.$$

To make a moral decision concerning the distribution of resources, one way is to consider the well-known Aristotle's principle of distributive justice (Moulin, 2003), "Equals should be treated equally, and unequals unequally, in proportion to the relevant *similarities* and *differences*." To achieve an *egalitarian* allocation, we utilize the notion of RCOF (Section 3.1). We update the objective to be the maximisation of the weighted sum of valuations of the movements. The weights of a movement i is given by the RCOF ρ_i that depends on the city connected by that movement i. The RCOF is designed to be higher for the flights connecting the cities with a lower value of SPI and a high population. The objective to maximise the weighted sum of valuations, i.e., $\operatorname{argmax}_A \sum_{j \in S} \sum_{i \in M} \rho_i\ v_{ij}\ x_{ij}$, therefore provides more opportunities to such cities.

We assume there is a constant cost g per unit of congestion. Our objective includes to minimise the total congestion cost, i.e. $\operatorname{argmin}_A \sum_{j \in S} \sum_{i \in M} e_j(x)\ g$, where, $e_j(x)$ (equation (2)) is a measure of the level of the congestion in slot j with respect to the allocation x. Combining the discussed goals we formulate the following objective function,

$$\operatorname*{argmax}_{A} \quad \sum_{j \in S} \left(\sum_{i \in M} (\rho_i\ v_{ij}\ x_{ij}) \quad - \quad e_j(x)\ g \right) \tag{4}$$

We call the combination of the two terms in expression (4) as *social utility.* Our objective is to maximize the social utility while maintaining the constraints. The expression for social utility captures the three *social* goals, namely, efficiency, egalitarianism, and congestion-awareness, and finds a balance between them.

In the following section, we introduce the proposed mechanism.

4 Proposed mechanism

The problem of *truthful* resource allocation has been studied extensively in mechanism design with transfers. The Vickrey-Clarke-Groves (VCG) payment rule (Vickrey, 1961; Clarke, 1971; Groves, 1973) has been one of the most celebrated mechanisms for truthful and *efficient* resource allocation among multiple rational agents. One limitation of VCG payment rule is that it applies only if the social goal is to achieve efficiency. In this paper, we consider a different objective of maximizing social utility (Equation (4)). Hence, VCG mechanism is not applicable for the implementation of this social goal. We consider a general form of VCG mechanism, known as the *affine maximizer* social choice function.

We propose a `E`galitarian and `C`ongestion `A`ware `T`ruthful `S`lot allocation (`ECATS`) mechanism as an ASF that consists of an allocation function $A(\cdot)$ and a payment function $p(\cdot)$. The allocation function uses the airlines' reported valuations for slots and the RCOF to find a socially egalitarian allocation where each movement is weighted with its RCOF. It also puts an additive penalty for congestion and maximizes this affine sum. The following integer linear program, therefore, computes the optimal allocation.

$$\begin{aligned}
\underset{A}{\text{argmax}} \quad & \sum_{j \in S} \left(\sum_{i \in M} (\rho_i \ v_{ij} \ x_{ij}) \quad - \quad e_j(x) \ g \right) \\
\text{subject to} \quad & \sum_{i \in M} x_{ij} \leqslant C_j \quad \forall j \in S, \\
& \sum_{j \in S} x_{ij} \leqslant 1 \quad \forall i \in M \\
& x_{ij} = \{0, 1\} \quad \forall i \in M \ \ \forall j \in S
\end{aligned} \tag{5}$$

The first term of the objective function multiplies each movement's valuation with their RCOFs (ρ_is) to equalize the opportunities to the flights to and from every city. The second term of the objective function subtracts the total congestion cost to cater to the congestion problem. The first constraint in the above optimization problem is the capacity constraint of each slot. The second constraint ensures that none of the movements are assigned to more than one slot.

Allocation The allocation function $A(\cdot)$ of `ECATS` computes the LP relaxation of IP (5) as follows.

$$\begin{aligned}
\underset{A}{\text{argmax}} \quad & \sum_{j \in S} \left(\sum_{i \in M} (\rho_i \ v_{ij} \ x_{ij}) \quad - \quad e_j(x) \ g \right) \\
\text{subject to} \quad & \sum_{i \in M} x_{ij} \leqslant C_j \quad \forall j \in S, \\
& \sum_{j \in S} x_{ij} \leqslant 1 \quad \forall i \in M \\
& x_{ij} \geqslant 0 \quad \forall i \in M \ \ \forall j \in S
\end{aligned} \tag{6}$$

In Section 5, we prove that the solution of the above LP will always be integral and therefore coincides with the solution of IP (5).

Payment The payment function $p(\cdot)$ in `ECATS` is given by,

$$p_i(v_i, v_{-i}) = \begin{cases} \frac{1}{\rho_i} \left(h_i(v_{-i}) - \Big(\sum\limits_{k \in M \setminus \{i\}} \rho_k \ v_k \left(\ A(v_i, v_{-i}) \ \right) - g \sum\limits_{j \in S} \ e_j(A(v_i, v_{-i})) \Big) \right) & \rho_i > 0 \\ 0 & \rho_i = 0 \end{cases} \tag{7}$$

where,

$$h_i(v_{-i}) = \sum_{k \in M \setminus \{i\}} \left(\rho_k \ v_k\big(A(v_{-i})\big) - g \sum_{j \in S} \ e_j(A(v_{-i})) \right)$$

The payment function for an airline i is proportional to the difference between the value of the optimal objective function when i is absent and present, respectively. The payment is inspired by the idea of marginal contributions in the affine maximizers (Roberts, 1979).

5 Theoretical results

In this section, we present the theoretical guarantees for the properties of ECATS. For better readability, some of the proofs are deferred to the appendix.

Our first result shows that under ECATS, none of the players can get better utility by misreporting her true information.

THEOREM 1. ECATS *is dominant strategy truthful.*

Proof. In terms of utilities, the theorem says,

$$u_i(f(v_i, v_{-i}), V) \geq u_i(f(v_i', v_{-i}), V), \quad \forall v_i, v_i' \in \mathbb{R}^{|n|} \;\; \forall i \in M \tag{8}$$

Let us assume for the contradiction that, there exist a movement i for which the corresponding airline has true valuations for the slots as, v_i, but misreports it as v_i'(the corresponding value function is v_i'), and gets better utility,

$$u_i(f(v_i', v_{-i}), V) > u_i(f(v_i, v_{-i}), V) \tag{9}$$

Suppose $\mathcal{A}(v_i', v_{-i}) = x'$ and $\mathcal{A}(v_i, v_{-i}) = x^*$. By definition of the utility function,

$$u_i(x^*, V) = v_i(x^*) - p_i(v_i, v_{-i})$$

$$= v_i(x^*) - \frac{1}{\rho_i}\Big(h_i(v_{-i}) - \Big(\sum_{k \in M\setminus\{i\}} \rho_k \; v_k(\; x^* \;) - g\sum_{j\in S} \; e_j(x^*)\Big)\Big)$$

$$= v_i(x^*) + \frac{1}{\rho_i}\Big(\sum_{k \in M\setminus\{i\}} \rho_k \; v_k(\; x^* \;) - g\sum_{j\in S} \; e_j(x^*)\Big) - \frac{1}{\rho_i}\Big(h_i(v_{-i})\Big)$$

$$= \frac{1}{\rho_i}\Big(\sum_{k \in M} \rho_k \; v_k(\; x^* \;) - g\sum_{j\in S} \; e_j(x^*)\Big) - \frac{1}{\rho_i}\Big(h_i(v_{-i})\Big) \tag{10}$$

and,

$$u_i(x', (v_i, v_{-i})) = v_i(x') - p_i(v_i', v_{-i})$$

$$= \frac{1}{\rho_i}\Big(\sum_{k \in M} \rho_k \; v_k(\; x' \;) - g\sum_{j\in S} \; e_j(x')\Big) - \frac{1}{\rho_i}\Big(h_i(v_{-i})\Big) \tag{11}$$

From inequality 9,

$$\frac{1}{\rho_i}\Big(\sum_{k \in M} \rho_k \; v_k(\; x' \;) - g\sum_{j\in S} \; e_j(x')\Big) - \frac{1}{\rho_i}\Big(h_i(v_{-i})\Big)$$
$$> \frac{1}{\rho_i}\Big(\sum_{k \in M} \rho_k \; v_k(\; x^* \;) - g\sum_{j\in S} \; e_j(x^*)\Big) - \frac{1}{\rho_i}\Big(h_i(v_{-i})\Big)$$

$$\sum_{k \in M} \rho_k \; v_k(\; x' \;) - g\sum_{j\in S} \; e_j(x') \;>\; \sum_{k \in M} \rho_k \; v_k(\; x^* \;) - g\sum_{j\in S} \; e_j(x^*)$$

The above inequality leads to the contradiction with the fact that x^* is the socially optimal allocation. And therefore, the best strategy for airlines is to report the valuations of their movements truthfully. $\square$

The above result implies that irrespective of the reported valuations of the other movements, a given movement's utility is maximized when it reports its valuations truthfully. Our next result shows that the movements are incentivized to participate in ECATS voluntarily.

THEOREM **2.** ECATS *is individually rational for every movement.*

Proof of Theorem 2. Consider the allocation given by $\mathcal{A}$ as x^*. The utility of the airline i is, $u_i((x^*, p(V)), V) = v_i(x^*) - p_i(V)\ \forall i \in M$. $v_i(\mathcal{A}(V)) \geq 0$, as it is the valuation by the allocation of slots. If $\rho_i = 0$ then $u_i((x^*, p(V)), V) \geq 0$. For the case when $\rho_i > 0$, the proof is as follows: by expanding the expression for p in the definition of utility, we get

$$u_i((x^*, p(V)), V) = v_i(x^*) - \frac{1}{\rho_i}\Big(h_i(v_{-i}) - \Big(\sum_{k\in M\setminus\{i\}} \rho_k\ v_k(x^*) - g\sum_{j\in S} e_j(x^*)\Big)\Big)$$

$$= v_i(x^*) - \frac{1}{\rho_i}\Big(h_i(v_{-i})\Big) + \frac{1}{\rho_i}\Big(\sum_{k\in M\setminus\{i\}} \rho_k\ v_k(\ x^*\ ) - g\sum_{j\in S}\ e_j(\ x^*\ )\Big)$$

$$= \frac{1}{\rho_i}\Big(\sum_{k\in M} \rho_k\ v_k(\ x^*\ ) - g\sum_{j\in S}\ e_j(\ x^*\ )\Big) - \frac{1}{\rho_i}\Big(\sum_{k\in M\setminus\{i\}} \rho_k\ v_k\big(\mathcal{A}(v_{-i})\big) - g\sum_{j\in S}\ e_j(\mathcal{A}(v_{-i})\ )\Big)$$

By adding and subtracting $\frac{1}{\rho_i}(v_i(\mathcal{A}(v_{-i})))$,

$$= \frac{1}{\rho_i}\Big(\sum_{k\in M} \rho_k\ v_k(\ x^*\ ) - g\sum_{j\in S}\ e_j(\ x^*\ )\Big) - \frac{1}{\rho_i}\Big(\sum_{k\in M} \rho_k\ v_k\big(\mathcal{A}(v_{-i})\big) - g\sum_{j\in S}\ e_j\big(\mathcal{A}(v_{-i})\big)\Big) + \frac{1}{\rho_i}(v_i(\mathcal{A}(v_{-i})))$$

$$= \underbrace{\Big(\sum_{k\in M} \rho_k\ v_k(x^*) - g\sum_{j\in S} e_j(x^*)\Big) - \Big(\sum_{k\in M} \rho_k\ v_k\big(\mathcal{A}(v_{-i})\big) - g\sum_{j\in S} e_j\big(\mathcal{A}(v_{-i})\big)\Big)}_{\geq 0} + \underbrace{v_i\big(\mathcal{A}(v_{-i})\big)}_{\geq 0}$$

The difference of first two terms is non-negative by the definition of x^*, the socially optimal allocation, and the third term is non-negative as it is the valuation by the allocation of slots, which proves that ECATS is individually rational. □

Next, we prove that ECATS is weakly budget-balanced; in other words, ECATS does not require an outside subsidy to operate.

THEOREM **3.** ECATS *satisfies weakly budget-balance.*

Proof. Let x^* denotes the optimal allocation by Equation (6). Suppose the mechanism is not weakly budget-balanced. Hence,

$$\sum_{i\in M} p_i(x^*) < 0$$

For the above inequality to be true, there must exist a movement $i \in M$ such that $p_i(x^*) < 0$. The $p_i(x^*)$ is computed as,

$$p_i(x^*) = \frac{1}{\rho_i}\left(h_i(v_{-i}) - \sum_{k \in M\setminus\{i\}}\left(\rho_k\ v_k(x^*) - g\sum_{j\in S} e_j(x^*)\right)\right)$$

Denote the optimal allocation when i is not present as $x^{'}$; $A(v_{-i}) = x^{'}$. If $p_i(x^*) < 0$ then by expanding the expression for $h_i(v_{-i})$ we get,

$$\frac{1}{\rho_i}\left(\sum_{k\in M\setminus\{i\}}\left(\rho_k\ v_k(x^{'}) - g\sum_{j\in S}\ e_j(x^{'})\right) - \sum_{k\in M\setminus\{i\}}\left(\rho_k\ v_k(x^*) - g\sum_{j\in S} e_j(x^*)\right)\right) < 0$$

The ρ_is are non-negative values hence,

$$\sum_{k\in M\setminus\{i\}}\left(\rho_k\ v_k(x^{'}) - g\sum_{j\in S}\ e_j(x^{'})\right) - \sum_{k\in M\setminus\{i\}}\left(\rho_k\ v_k(x^*) - g\sum_{j\in S} e_j(x^*)\right) < 0 \tag{12}$$

Without any movement i, any solution to the mechanism with all agents remains feasible and has positive value. Hence, the inequality (12) contradicts with the definition of x' which proves that `ECATS` satisfies weakly budget-balance. □

The following few results show that even if the allocation problem of IP (5) falls in a computationally intractable class, its special structure can be used to find a tractable solution.

THEOREM **4.** *The allocation of* `ECATS` *given by LP (6) has an integral optimal solution and is polynomially solvable.*

Proof. Using the definition of e_j in Equation (2), LP (6) can be written as follows

$$\begin{aligned}
\underset{A,w}{\text{argmax}} \quad & \sum_{j\in S}\left(\sum_{i\in M}(\rho_i\ v_{ij}\ x_{ij}) \quad - \quad w_j\ g\right) \\
\text{subject to} \quad & \sum_{i\in M} x_{ij} \leqslant C_j, \quad \forall j \in S \\
& \sum_{j\in S} x_{ij} \leqslant 1, \quad \forall i \in M \\
& \sum_{i\in M} x_{ij} - w_j \leqslant T_j, \quad \forall j \in S \\
& x_{ij} \geqslant 0, \quad w_j \geqslant 0, \quad \forall i \in M, \quad \forall j \in S
\end{aligned} \tag{13}$$

where, $T_j = C_j(1-\lambda)$.

CLAIM **1.** *The coefficient matrix of optimization problem (13) is totally unimodular (TU).*

Proof. First we linearize the variables x and w of the optimization problem into a single vector $\bar{x}$ as $(x_{11}, x_{12}, \ldots, x_{1n}, \ldots, x_{m1}, x_{m2}, \ldots, x_{mn}, w_1, w_2, \ldots, w_n)^\top$. The constraints in the Equation (13) can be written as $Z\bar{x} \leqslant b$, where, $Z_{(m+2n)\times(mn+n)}$ is the coefficient matrix and

$b_{(m+2n)\times 1}$ is the bound vector as shown below.

$$\begin{bmatrix} I_{n\times n} & I_{n\times n} & I_{n\times n} & \dots & I_{n\times n} & 0 \\ \mathbb{1}_{1\times n} & 0 & \dots & 0 & 0 & 0 \\ 0 & \mathbb{1}_{1\times n} & \dots & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbb{1}_{1\times n} & 0 \\ I_{n\times n} & I_{n\times n} & I_{n\times n} & \dots & I_{n\times n} & -I_{n\times n} \end{bmatrix} \begin{bmatrix} x_{11} \\ \vdots \\ x_{1n} \\ \vdots \\ x_{m1} \\ \vdots \\ x_{mn} \\ w_1 \\ \vdots \\ w_n \end{bmatrix} \leqslant \begin{bmatrix} C_1 \\ \vdots \\ C_n \\ 1 \\ \vdots \\ 1 \\ T_1 \\ \vdots \\ T_n \end{bmatrix}$$

The first n rows of Z has m $(n \times n)$ identity matrices followed by $(0)_{n\times n}$. Each of the next m rows has exactly one $(1 \times n)$ vector of all 1s staggered as shown above. The last n rows are similar to the first n rows with the last n columns being a negative $n \times n$ identity matrix.

We use the Ghouila-Houri (GH) characterization (Ghouila-Houri, 1962; de Werra, 1981) to prove that Z is TU, which says that a matrix $Z_{p\times q}$ is TU if and only if any subset R of rows, $R \subseteq \{1, 2, \cdots, p\}$, can be partitioned into two subsets R_1 and R_2, such that, $\sum_{i\in R_1} z_{ij} - \sum_{i\in R_2} z_{ij} \in \{1, 0, -1\}$, where $j = 1, 2, \dots, q$. In our case, each column of the coefficient matrix Z has at most three 1 or one -1. Note that, the rows can be easily partitioned into *three* classes: class 1 consists of first n rows, class 2 consists of next m rows, class 3 consists of last n rows. There are three exhaustive cases for the subset R of rows of Z.

1. R consists of rows from all three classes of rows.
2. R consists of rows from any two classes of rows.
3. R consists of rows from exactly one class of rows.

For case 1, we find the two disjoint subsets R_1 and R_2 of R in an iterative manner. Begin with the partition where R_1 consists of rows from the first two classes of rows of Z in R, and R_2 consists of rows from last class rows of Z in R. There can be a situation where for a certain column j, the sum $\sum_{i\in R_1} z_{ij} = 2$ and the $\sum_{i\in R_2} z_{ij} = 0$. This can only happen when for column j and rows in R_1, the sum of z_{ij}s where i's are from the rows from class 1 is 1, for the i's in class 2 there exists a row that has n 1s intersecting with j, and the sum of z_{ij}s where i's are from the rows in R_2 has no 1s in j. In such a case, we move that row in class 2 from R_1 to R_2. Repeat this procedure for every such column until no such situation exists. This procedure is guaranteed to converge to a partition of R such that GH conditions are met.

Cases 2 and 3 are straightforward. In case 2, the partition R_1 and R_2 are the intersections of R with the respective classes of rows. For case 3, any partition of R satisfies GH conditions. Hence proved. □

The optimization problem in Equation (13) has a TU coefficient matrix, which implies LP (6) yields an optimal solution in integers and is solvable in polynomial time. □

Theorem 4 shows that the solution to the LP relaxation of the allocation problem is without loss of optimality. LPs are known to be polynomially computable. However, in general, it can be weakly polynomial, i.e., the space used by the mechanism may not be bounded by a polynomial in the input size. The forthcoming results show that the solutions of allocation and payments

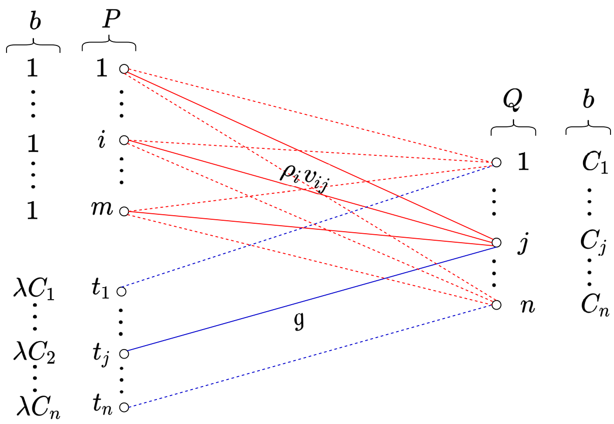

Figure 1: Constructed edge-weighted bipartite graph $G = (P, Q, E)$ and b

of ECATS are strongly polynomial. To show this, we reduce (LP 13) to the b-matching problem, which is known to be strongly polynomial (Anstee, 1987).

Definition 6 (b-Matching Problem (Tamir and Mitchell, 1998)). *Consider a graph $G = (V, E)$, where V is the set of nodes and E is the set of edges. Each edge $e_{u,v} \in E$ between any two nodes $u, v \in V$, has a cost $c_{u,v}$. Let $b = (b_1, b_2, \ldots, b_{|V|})$. A b-matching problem for G is to find the non-negative integer edge weights $w_{u,v}$ which maximises the total cost, $\sum_{u,v \in V} c_{u,v}\ w_{u,v}$ where the sum of weight on edges incident to a node u is no more than b_u, $\forall u \in V$.*

Theorem 5. *The allocation of* ECATS*, given by LP (6), is implementable in strongly polynomial time.*

Proof. Consider an edge weighted bipartite graph, $G = (P, Q, E)$, where $P = M \cup \{t_j | j \in S\}$ and $Q = S$. The edges in E are in two disjoint partitions, E_1 and E_2. The first partition is $E_1 = \{(e_{i,j}) | i \in M \text{ and } j \in Q\}$. Set the cost $c_{i,j} = \rho_i v_{ij}, \forall e_{i,j} \in E_1$, where ρ_i and v_{ij} are the RCOF and valuation of movement i for slot j respectively. The other partition is $E_2 = \{e_{t_j,j} | t_j \in P \setminus M \text{ and } j \in Q\}$ having the cost $c_{t_j,j} = g$, where g is per unit congestion cost. Define b as, $b_i = 1$ for every $i \in M$, $b_j = C_j$ for every $j \in Q$ and for every $t_j \in P \setminus M$, $b_{t_j} = \lambda C_j$[9]. Figure 1 shows the edge-weighted bipartite graph $G = (P, Q, E)$. The red and blue colored edges denote the subsets E_1 and E_2 respectively. To make the notations simpler, we represent $e_{i,j}$ as (i, j). The objective function of the b-matching problem over G (with y being the optimization variables) is

$$\sum_{j \in Q} \sum_{i \in M} \rho_i v_{ij}\ y_{i,j} + \sum_{j \in Q} g\ y_{t_j,j}$$

with the constraints, $\sum_{q \in Q} y_{p,q} \leqslant b_p$ and $\sum_{p \in P} y_{q,p} \leqslant b_q$, $\forall p \in P$, $\forall q \in Q$. By the definition of b-matching problem, the optimal solution y^* of b-matching of G has,

$$y^*_{t_j,j} = \begin{cases} C_j - \sum_{i \in M} y^*_{i,j} & \sum_{i \in M} y^*_{i,j} - C_j(1-\lambda) > 0 \\ \lambda C_j & \sum_{i \in M} y^*_{i,j} - C_j(1-\lambda) \leqslant 0 \end{cases} \tag{14}$$

The correctness of the above equation can be proved as follows. First, note that for an optimal solution y^*, if $\exists j \in S$ with $\sum_{i \in M} y^*_{ij} < C_j$ then $\nexists i \in M$ such that $y^*_{ij} < b_i$ and $\rho_i v_{ij} \geqslant g$ – because otherwise y^*_{ij} can be increased maintaining the feasibility of the constraints ($b_i = 1$

[9] This is the size of congestion prone capacity of slot j.

and $b_j = C_j$) which will increase the value of the objective function, contradicting the optimality of y^*.

Consider case 1 of Equation (14), $\sum_{i\in M} y^*_{ij} - C_j(1-\lambda) > 0$: $\sum_{i\in M} y^*_{i,j}$ is the total weight on the red edges. The remaining weight $C_j - \sum_{i\in M} y^*_{i,j}$ is less than λC_j and, as $\nexists i \in M$ such that $y^*_{ij} < b_i$ and $\rho_i v_{ij} \geqslant g$ therefore, adding all the remaining weight to $y^*_{t_j,j}$ is always (a) within the constraints of b-matching and (b) maximize the value of the objective function.

Consider case 2 of Equation (14), $\sum_{i\in M} y^*_{ij} - C_j(1-\lambda) \leq 0$: the remaining weight $C_j - \sum_{i\in M} y^*_{i,j}$ is more than λC_j. But, $y^*_{t_j,j}$ can not be greater than λC_j as $b_{t_j} = \lambda C_j$. The optimal solution is to add the remaining weight to $y^*_{t_j,j}$ within the constraint $b_{t_j} = \lambda C_j$ for j.

Our next result formally reduces the allocation problem of ECATS into a b-matching problem. We do that via a lemma, that constructs a solution x for our problem from the solution y of b-matching of G as, $x_{i,j} = y_{i,j}$, for $i \in M$ and $j \in S$. Similarly, we construct a solution y for b-matching of G from a solution x for our problem as, $y_{i,j} = x_{i,j}$, for $e_{i,j} \in E_1$ and, $y_{t_j,j}$ using Equation (14) for $e_{t_j,j} \in E_2$.

Lemma 1. *Let y^* is a solution for b-matching for graph G and x^* is s.t. $x^*_{i,j} = y^*_{i,j}$, $\forall i \in M$ and $\forall j \in Q$. Then, x^* is the optimal solution for the LP in Equation (6) iff y^* is an optimal solution for b-matching for graph G.*

Proof. First, we prove that if y^* is an optimal solution for b-matching for graph G, then x^* is an optimal solution for the LP in Equation (6).

Suppose the above lemma is not true and the optimal solution for LP in Equation (6) is $\bar{x}$ but not x^*. Then, for the value of objective function in Equation (6),

$$\sum_{j\in S}\left(\sum_{i\in M}(\rho_i\ v_{ij}\ \bar{x}_{i,j})\ -\ e_j(\bar{x})\ g\right) > \sum_{j\in S}\left(\sum_{i\in M}(\rho_i\ v_{ij}\ x^*_{i,j})\ -\ e_j(x^*)\ g\right)$$

using the definition of $e_j(x)$ from Equation (2),

$$\sum_{j\in S}\left(\sum_{i\in M}\rho_i\ v_{ij}\ \bar{x}_{i,j}\right) - g\sum_{j\in S}\max\left(\sum_{i\in M}\bar{x}_{i,j} - C_j(1-\lambda),\ 0\right)$$
$$> \sum_{j\in S}\left(\sum_{i\in M}\rho_i\ v_{ij}\ x^*_{i,j}\right) - g\sum_{j\in S}\max\left(\sum_{i\in M}x^*_{i,j} - C_j(1-\lambda),\ 0\right)$$

We divide the set of slots S in two disjoint subsets S_1, S_2 with respect to $\bar{x}$ such that, $S_1 = \{j|\sum_{i\in M}\bar{x}_{i,j} - C_j(1-\lambda) > 0\}$ and $S_2 = \{j|\sum_{i\in M}\bar{x}_{i,j} - C_j(1-\lambda) \leqslant 0\}$. Similarly, divide S in two disjoint subsets S_3, S_4 with respect to x^* such that, $S_3 = \{j|\sum_{i\in M}x^*_{i,j} - C_j(1-\lambda) > 0\}$ and $S_4 = \{j|\sum_{i\in M}x^*_{i,j} - C_j(1-\lambda) \leqslant 0\}$.

$$\sum_{j\in S}\left(\sum_{i\in M}\rho_i\ v_{ij}\ \bar{x}_{i,j}\right) + g\sum_{j\in S_1}\left(C_j - \lambda C_j - \sum_{i\in M}\bar{x}_{i,j}\right)$$
$$> \sum_{j\in S}\left(\sum_{i\in M}\rho_i\ v_{ij}\ x^*_{i,j}\right) + g\sum_{j\in S_3}\left(C_j - \lambda C_j - \sum_{i\in M}x^*_{i,j}\right)$$

Adding $g \sum_{j \in S} \lambda C_j$ on both sides of the above inequality,

$$\sum_{j \in S} \left(\sum_{i \in M} \rho_i \ v_{ij} \ \bar{x}_{i,j} \right) \ + \ g \sum_{j \in S_1} \left(C_j - \sum_{i \in M} \bar{x}_{i,j} \right) + \ g \sum_{j \in S_2} \lambda C_j$$
$$> \sum_{j \in S} \left(\sum_{i \in M} \rho_i \ v_{ij} \ x^*_{i,j} \right) \ + \ g \sum_{j \in S_3} \left(C_j - \sum_{i \in M} x^*_{i,j} \right) + \ g \sum_{j \in S_4} \lambda C_j$$

Let $\bar{y}$ is a solution of b-matching for G corresponding to the solution $\bar{x}$ of LP in Equation (6), then the expression at right side in above inequality is the value of objective function for b-matching problem, where $\forall j \in S_1$ the case 1 of Equation (14) is true and $\forall j \in S_2$ case 2 of Equation (14) is true. The above inequality implies that the value of the objective function of the b-matching problem for solution $\bar{y}$ is more than that for y^*. Therefore, the above inequality contradicts with y^* being the optimal solution of the b-matching for graph G, which implies that x^* is the optimal solution for the LP in Equation (6).

To prove the other direction of Lemma 1, we construct y^* from the optimal solution x^* for the LP in Equation (6) as, $y^*_{i,j} = x^*_{i,j}$, for $e_{i,j} \in E_1$ and, $y^*_{t_j,j}$ using Equation (14) for $e_{t_j,j} \in E_2$. The proof follows by similar argument in the reverse order. □

Lemma 1 shows that for every instance of the LP in Equation (6), there exists an instance of b-matching problem such that the optimal solution of that instance of b-matching problem gives the optimal solution for LP in Equation (6). Therefore, LP in Equation (6) is solvable in strongly polynomial time as there exists a combinatorial, strongly polynomial algorithm to solve the b-matching problem (Anstee, 1987). □

As the computation of payment for an airline $i \in M$ requires computation of a socially optimal allocation $\forall j \in M \setminus \{i\}$ using the LP in Equation (6), we get Corollary 1.

Corollary **1.** *The computation of payments for all the airlines is implementable in strongly polynomial time.*

Following the Theorem 5 and Corollary 1, we get Theorem 6.

Theorem **6.** *There is a combinatorial strongly polynomial time algorithm for computing the allocation and payments $f = (A, p)$ in* `ECATS`*.*

Discussion Our analysis here assumes that a movement is possible to be scheduled in any slot. The cases of infeasible slots for a movement can be accommodated in this model by assigning a large negative valuation for those slots such that the allocation mechanism never picks such slots. There is, however, one additional constraint that involves a minimum gap in the schedule of two movements that are served via the same aircraft. Generally, movements are paired, i.e., every arriving movement also has a complementary departing movement and the allocation needs to have a minimum time gap between these two movements. This gives rise to a sufficiently complicated optimization problem as presented in Appendix A. We are able to show that the properties like truthfulness, egalitarianism, congestion reduction, and weak budget balance properties will continue to hold in this modified scenario as well. We have also experimentally checked that the computational complexity is polynomial. However, the rigorous proof of tractability is left as a future exercise. The detailed discussion of this extension is presented in Appendix A.

6 Experimental results

In this section, we investigate the performance of ECATS in real-world scenarios. While ECATS satisfies several desirable properties of a slot allocation mechanism, its performance with varying congestion costs, use of remote city opportunity factor (RCOF) and relevance are not theoretically captured, calling for an experimental study.

For the experiment, we obtain data from two airports in India (Indira Gandhi International Airport (DEL) and Chennai International Airport (MAA)). DEL and MAA are major hub airports and the busiest in India. In 2018, DEL managed approximately 70 million passengers and was the 12^{th} busiest airport in the world and the sixth busiest in Asia. It is designated as a level 3 airport and has three near-parallel runways. The MAA has a handling capacity of 22.5 million passengers and is the 49^{th} busiest airport in Asia. Both airports are coordinated[10]; however the traffic movement in DEL is almost four times that of MAA. The purpose of choosing these two airports is to evaluate the performance of ECATS for the allocation and payments under different demand/congestion profiles.

We also compare ECATS with (a) the *Current allocation*, i.e., the current movement to slot allocation, and (b) the PSAM proposed by Ribeiro et al. (2018) that complies with the *IATA guidelines*. The AAI monitors the current allocation where the coordination process is initiated when the coordinator provides each airline with the details of their historical slots. The slots are allotted according to grandfathering rights, where priority is given to historic slots. The PSAM model optimizes slot allocation decisions based on slot availability and airline slot requests while accounting for various preferences and requirements in the IATA guidelines. It minimizes the number of slot requests rejected or displaced. The results show that slot allocation significantly reduces flight displacement to accommodate all requests.

6.1 Data collection and processing

We collect the flight schedule data with landing and take-off slot details, city of arrival/departure, flight number, and service provider. We obtain the flight movement data[11] from January 21 to 25, 2020. An average of 867 movements occurred daily for DEL and 220 for MAA.[12] Based on the data, we obtain the input dataset of a single day and apply ECATS for experimental analysis. The details of obtaining a single-day dataset are as follows.

6.1.1 Slot capacities

We found ample literature where researchers have imposed constraints on 15-min intervals to allocate airport resources (Ribeiro et al., 2018; Zografos et al., 2012; Cheung et al., 2021). The AAI and IATA have also recognized slots to be a 15 min intervals. In line with the practice and literature, we divide the 24h of each day into slots (15-min time intervals) for each day of the collected data.

Obtaining the airport's exact capacity parameter is an active research area. Airports' capacity depends on available resources, weather conditions, and many other factors. Airports can also declare capacity based on historical data. Therefore, several methods have been proposed to have a quantifiable measure. The existing allocation methods consider the capacities estimated

[10]Coordinated airports are the ones where landing airlines have to acquire landing rights and ensure their operation during a specific time period. The slots are administered by the airport operator or a government aviation regulator. Landing/takeoff demand at these airports exceeds its capacity.

[11]https://www.flightradar24.com

[12]We chose these dates to obtain the normal air movement patterns before COVID-19 and also because the data were most detailed during this period.

by airport authorities. Since, we did not have access to such data, we estimated the capacity for this experiment using the historical number of movements for each slot in an airport. The capacity is estimated in two steps: (i) we consider the maximum number of movements (arriving or departing) in *a specific slot* for five consecutive days (mentioned before), and (ii) we take the average of those maxima to find airport's capacity. This capacity is treated as the capacity of *each* slot of that airport. We want to emphasize that even though this is a specific heuristic to produce the slot capacity (which we have chosen for reasons explained below), none of the central results, e.g., truthfulness, and social utility, is affected by this choice. We provide an example to illustrate the steps better. Suppose, the 9:00 to 9:15 AM slot has 15, 12, 14, 15, and 16 movements in the five consecutive days we collected the data. We take the maximum, i.e., 16, at the end of step i, and take the average of those maxima for all the 24×4 slots of a day as the capacity of each airport slot. A natural question might arise as to why we did not take the maximum of all such maxima for the slot capacity. In such a case, the capacities will be so high that none of the movements in our dataset will ever witness congestion (since we are calculating the capacity from the same dataset).

We divide the airport's slot capacity into two parts by setting a threshold of $(1-\lambda)$ fraction of the slot capacity. We consider the slot congestion-free if the number of movements is less than or equal to this threshold, and congestion-prone otherwise. For the experiments, we assume $\lambda = 0.2$.

6.1.2 Valuations

The movement valuations are the private information of the airlines and are not publicly shared. The movement valuation for a slot is the cumulative value the slot gives over six months. Hence, the valuation can be estimated by computing the *revenue* from one-time travel multiplied by the number of times the flight service in the six months in that slot.

To estimate the revenue earned by the airline for an origin-destination pair and a slot, we require three parameters: the ticket prices, the aircraft's capacity, and the average load factor[13]. We use the following expression to compute the revenue:

$$\begin{aligned}\text{Revenue earned by a movement} = {} & \text{average of the ticket fare of the movement across five days} \\ & \times \text{ average load factor of the movement} \\ & \times \text{ capacity of the aircraft used by the movement.}\end{aligned}$$

For example, if an Indigo flight (movement) operates A320neo, with a seating capacity of 194, from DEL to Mumbai airport (BOM) at 09:15 with an average ticket fare of Indian Rupees (₹) $3,000$ and a load factor of 90%, the revenue for that movement, in the slot 09:15, will be ₹$3,000 \times 194 \times 0.9 =$ ₹$523,800$.

Since airlines use dynamic pricing, we have considered average fares. Moreover, our theoretical guarantees are independent of average fare calculations. The ticket price data were collected from a booking website[14] for five consecutive days. For the aircraft capacity and load factor data of different destinations, we refer to the annual financial results of two public airlines (Indigo and SpiceJet).

However, the revenue obtained in the above steps provides us with the data points only for those slots in which the flight operates, because we can access the ticket fares from the booking websites only for the current allocation. Using an example, We explain the procedure to fill in the missing data points. Let a movement i operate only at slot 9:00-9:15. Then, to

[13]The load factor is a metric to measure the percentage of available seating capacity filled with passengers.

[14]www.ixigo.com

estimate the revenue of i at other slots (say 9:45-10:00), we consider the revenue of the other operating flights in the slot 9:45-10:00 because we have already estimated their revenue for slot 9:45-10:00. We generate a histogram of the revenues of all such flights, which functions as an empirical distribution for the revenues earned by the movements in that slot. Then, we uniformly randomly choose a value from the histogram and fix it as the revenue of i for slot 9:45-10:00. We follow these steps for each slot for which we do not have the ticket fare in the collected data. This way, we estimate the revenue of every movement for each slot of the day, capturing the ticket fare variation across different time slots.

In the experiments, we have an input dataset for a single day. Therefore, we consider the valuation of a movement for a slot as equal to the revenue earned by that movement in that slot of a day. In other words, to compute the valuation, we do not multiply the number of times the flight services in the next six months in that slot. For consistency, the payments calculated using the mechanism are also only for a single day. Notice that the experimental analysis remains consistent even after valuations and payments are scaled up by multiplying by a constant. This result is because the mechanism is applied per day for a week, and the same allocation of a week is replicated for the next six months. Since the ticket prices are in Indian Rupees (INR), the valuation unit is also INR.

Note that the experimental setup will remain unchanged even if we generate $v_{ij}s$ using other data points or methods. The valuation estimated for the experiment in this study ensures that the results can be related to a real-world airport operation. The model's properties including truthfulness and improvement in social and individual utilities, are unaffected by the choice of the valuations and will hold for any airport with a different air traffic volume and valuations of any kind. Therefore, this experiment's valuations are not too tightly dependent on the estimated data valuations. Also, the timing of data collection does not affect the experimental setup because the effect will be similar for all movements.

6.1.3 RCOF

To calculate the RCOF (Equation (3)), we require two types of data:

1. The social progress index (SPI) of the origin/destination cities.

2. The population of the cities.

The Institute for Competitiveness, India[15] provided the SPI data. The population data is obtained from the Office of the Registrar General and Census Commissioner, India[16]. We keep the parameter $\alpha = 0.2$ to compute the RCOF for the experiments.

6.1.4 Summary of data

On plotting the average daily movements of the two airports (see Figure 2), we observe that flight movements are approximately four times higher in DEL than in MAA. The flight schedule has a higher variation under the current allocation, with a few time slots having numerous allocated movements, and `ECATS` provides a comparatively uniform flight movement with fewer fluctuations. A stable air traffic movement minimizes operational quality losses and improves airport resource allocation. Airports witness situations when terminals are clogged because of the misallocation of airport resources—in other words, avoiding traffic fluctuations and having a stable traffic movement will aid in the optimal use of airport resources. Incorporating congestion cost is vital in the evenness of slot allocations, discussed in the following subsection.

[15] https://competitiveness.in

[16] https://censusindia.gov.in

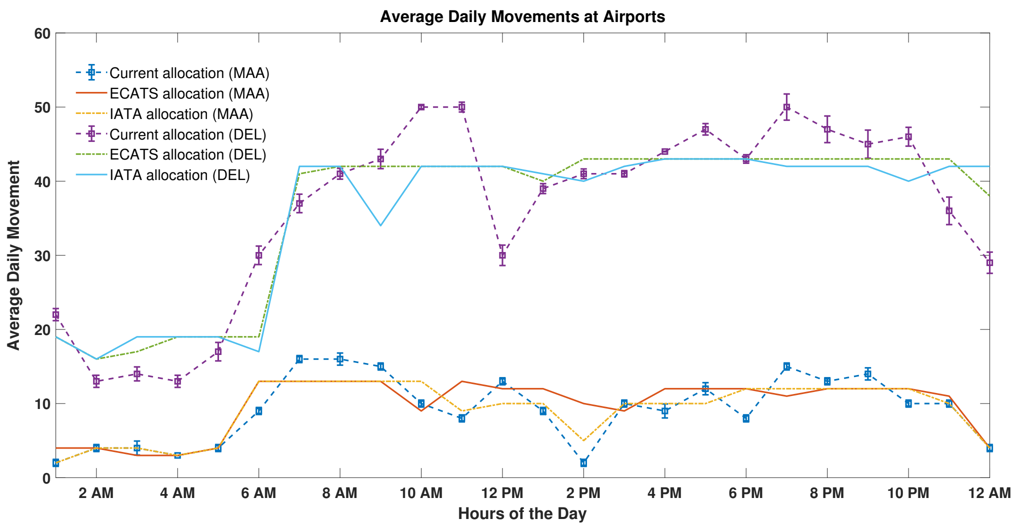

Figure 2: Average daily movements in the two airports.

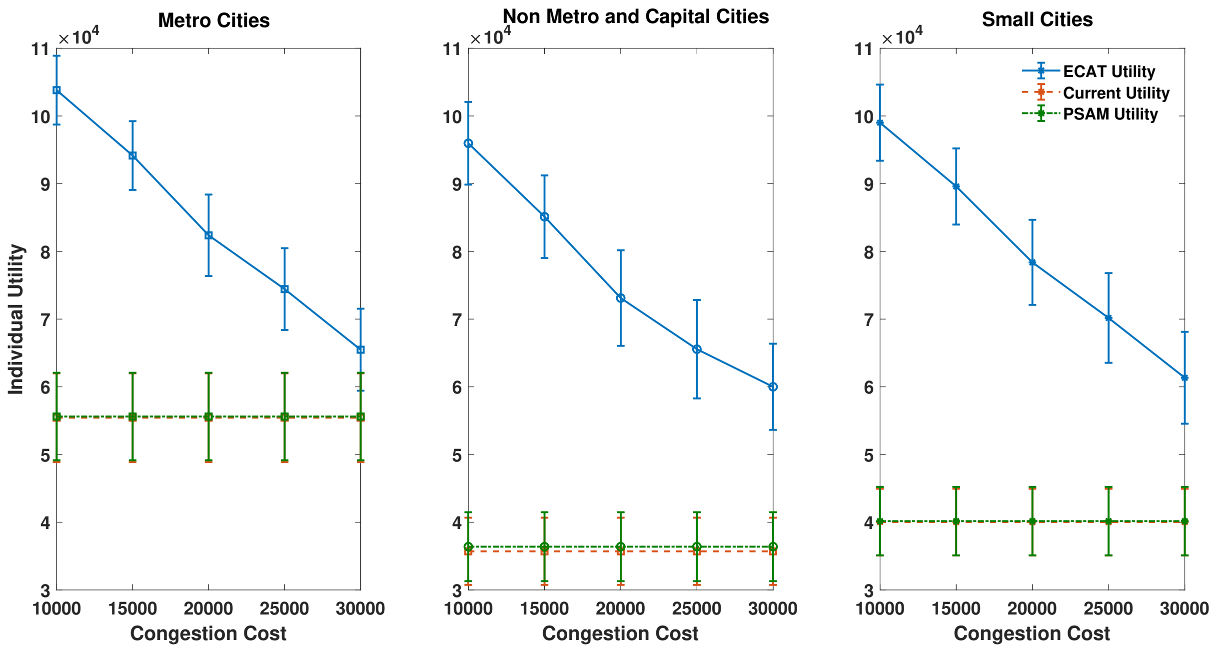

Figure 3: Individual utility of the movements at Delhi (DEL) airport.

6.2 Effect of congestion costs on individual and social utilities

We study the effect of congestion costs at individual and social levels. We define the individual utility as the sum of utilities (i.e., the difference of valuation and payment for each movement) of each movement divided by the total number of allotted movements. The social utility is the value of the objective function in the optimization problem given by Equation (5). Note that since the valuations are in INR, the unit of the utilities is also INR. We find that the individual utility decreases with congestion costs. However, the utility is still significantly higher than the other two mechanisms (Figures 3 and 4). Furthermore, each movement's utility is positive even with the increasing congestion cost, which signifies that the airlines stand to gain by participating in the proposed mechanism.

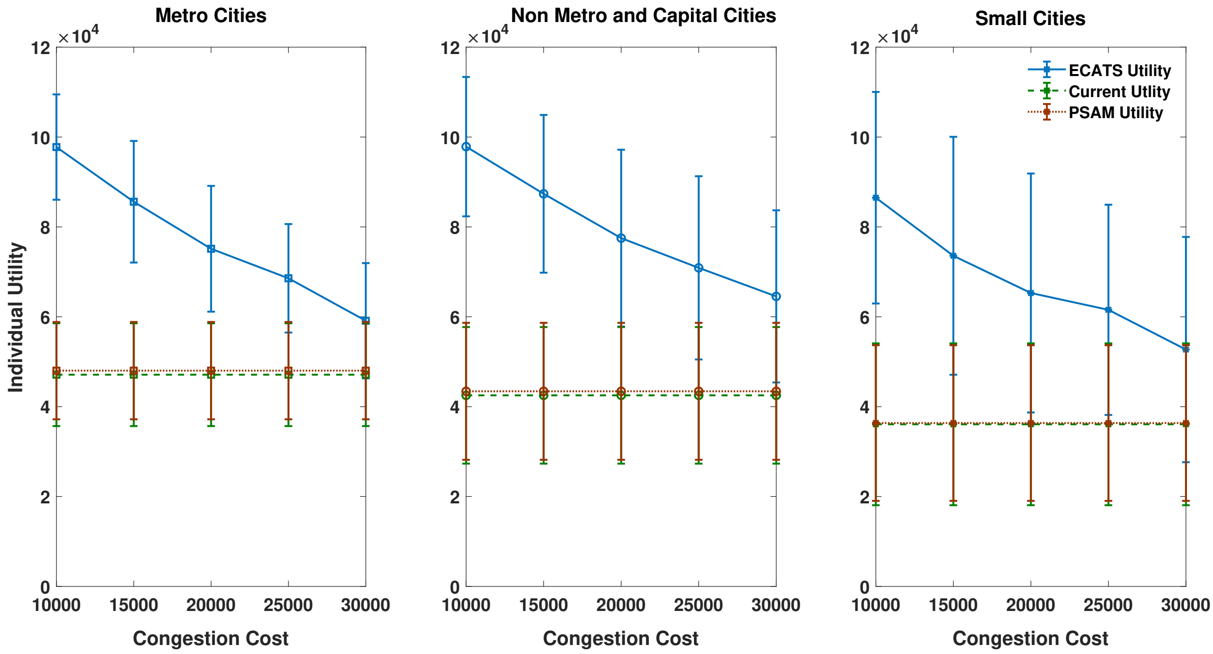

Figure 4: Individual utility of the movements at Chennai (MAA) airport.

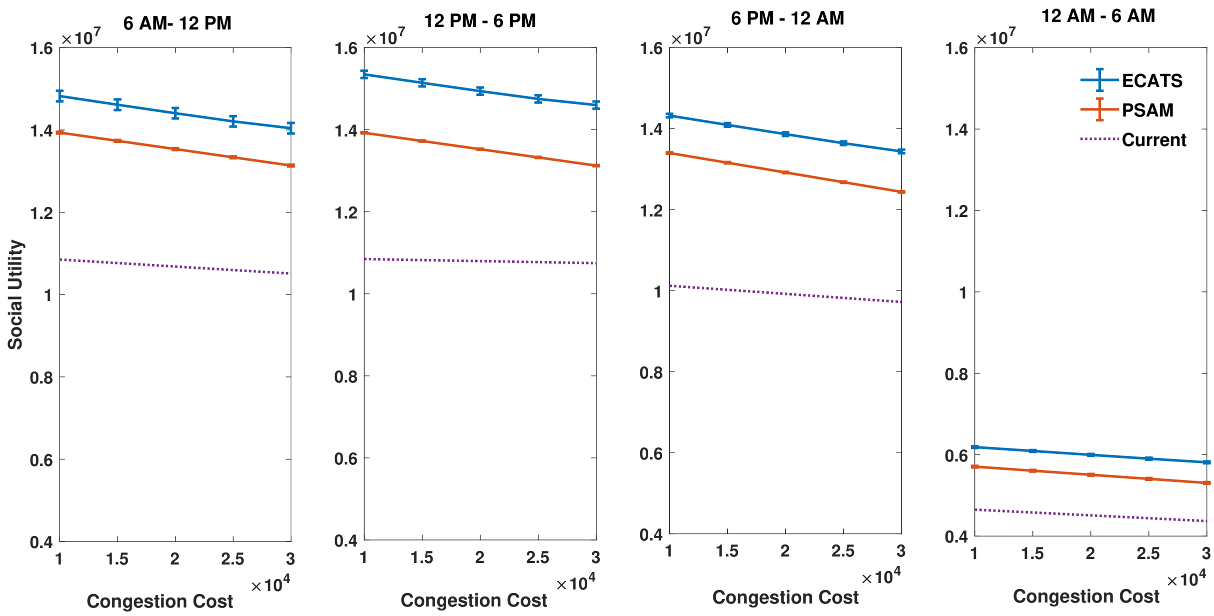

Figure 5: Social utility versus congestion cost: Delhi airport.

We also observe that the individual utility is comparable for the case of different connection types (metro, capital, and remote cities). The result shows airlines will gain similar utility by operating in trunk routes or connecting to remote cities. The individual utility for ECATS decreases but is unchanged for the other two mechanisms because there was no direct consideration of valuation maximization and congestion costs for slot allocations in the PSAM and current allocations.

Figures 5 and 6 show that the social utility decreases with the increase in congestion costs. As the congestion cost increases, ECATS considers the trade-off between allocating slots above the threshold capacity (and increasing congestion) and rejecting the slot request. Only move-

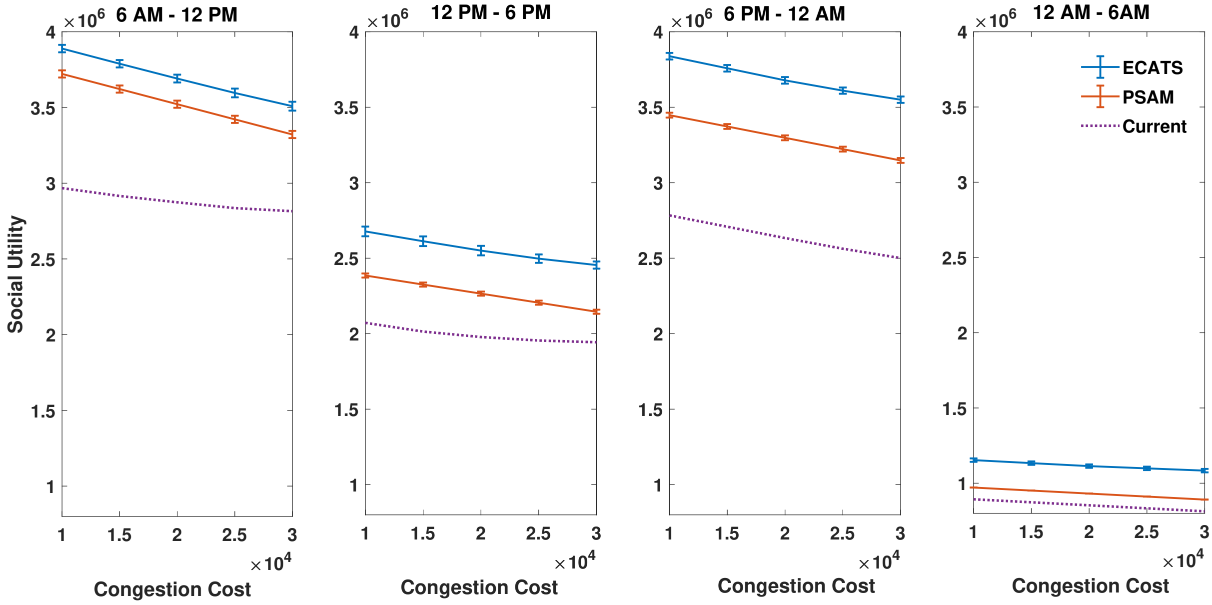

Figure 6: Social utility versus congestion cost: Chennai airport.

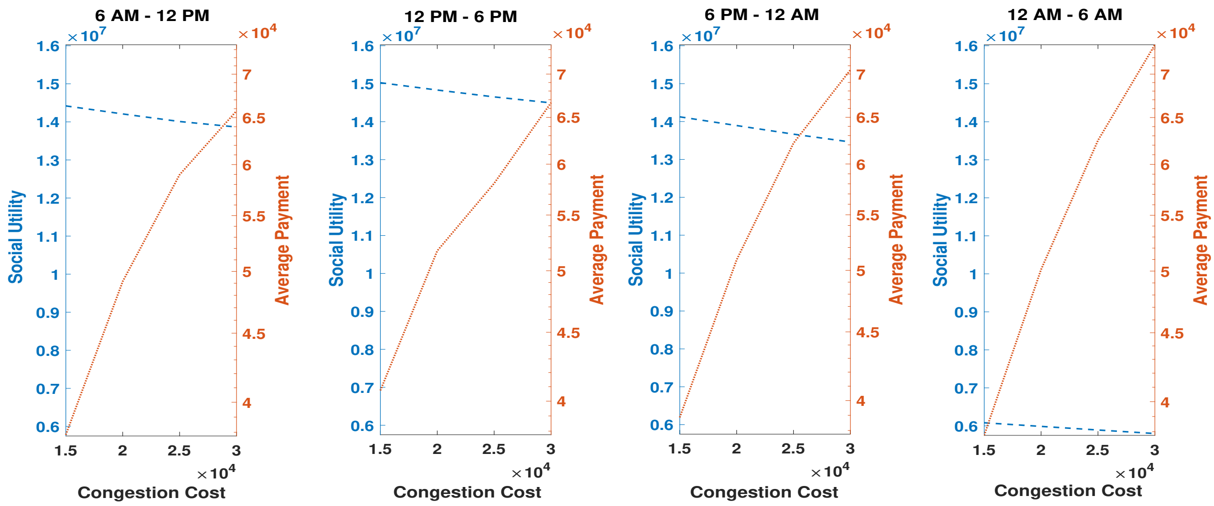

Figure 7: Social utility and average payment versus congestion cost: Delhi airport.

ments with high valuations are allocated to these slots, and those with low values are rejected. We see that the social utility of ECATS is higher than PSAM because the latter focuses on minimizing the displacement of the requested slot and does not consider the congestion cost at the time of allocation. We divide the 24h into four intervals of 6h each for better analysis and representation. We observe that even for time intervals from 12 to 6 AM, where valuations are the lowest, ECATS performs better than PSAM and the current allocation. Also, the social utility is comparable across different time intervals and shows a similar trend in all cases. These experiments demonstrate that a value-sensitive allocation like ECATS can improve both the individual and social utility by a significant amount.

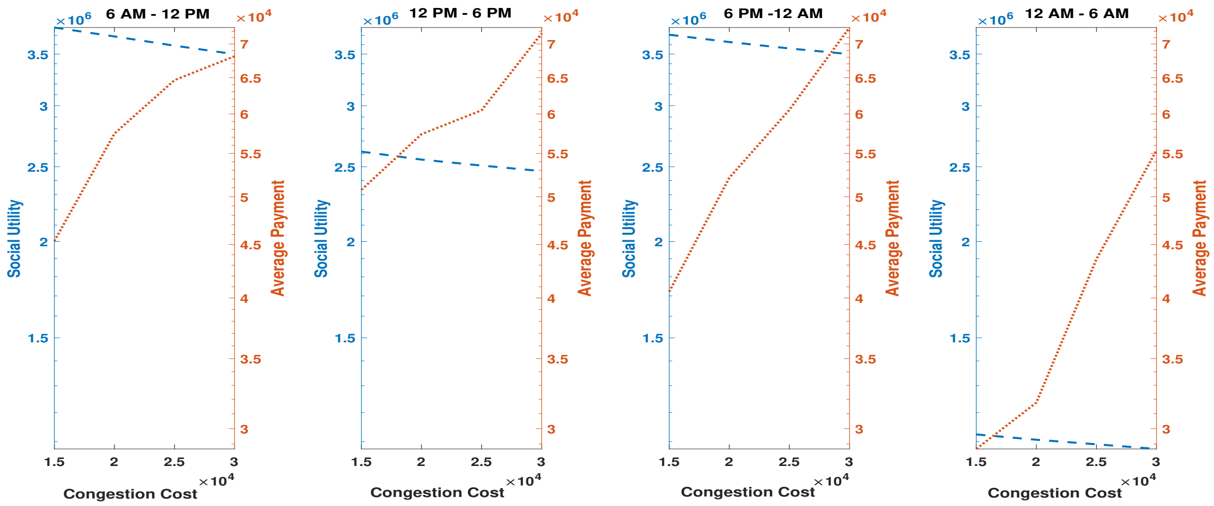

Figure 8: Social utility and average payment versus congestion cost: Chennai airport.

6.3 Effect of congestion costs on payments

One of the main features of `ECATS` is the payment function making mechanism truthful. In this section, we study the effect of congestion costs on the resulting average payments of the movements that are allocated a slot. Each movement's payment is calculated using Equation (7). Figures 7 and 8 show an uptrend of average payments, which is expected because a higher congestion cost introduces more competition among the movements for the slots. The social utility is also plotted in these figures to reference the payment scales. Figure 7 and 8 depict the increasing payment with congestion costs and the decreasing social utility, whereas Figures 5 and 6 compare the social utility across different mechanisms.

6.4 RCOF and its impact

The flights to metro cities and financial hubs have a high valuation since the ticket prices and load factors are high, giving them an advantage if the allocation is solely based on valuation. The ticket fare and demand for remote routes are comparatively less due to their low purchasing power or the cap imposed by government intervention. Hence, we provide equal opportunities to remote cities, using the RCOF from Equation (3) in the slot allocation process. It assigns a higher value to cities with a low SPI and a considerable population to benefit from air connectivity. Since the RCOF is multiplied by the valuations of the movements in the objective function of Equation (5), the chances of allocation of the remote cities in larger airports increase and simultaneously, the factor $1/\rho_i$ in the payment function leads to a lower payment for such flights.

We observe that the individual utilities of the flights to and from remote cities are comparable to the metro cities despite their low valuations (Figures 3 and 4), for Delhi and Chennai airports. This happens because the average slot payment for metro cities is higher than in non-metro state capitals, which is higher than in remote cities. Hence, `ECATS` is more egalitarian among the flights irrespective of the origin/destination.

From the regulator's viewpoint, `ECATS` offers opportunities for flights from remote cities and provides an incentive for airlines to operate in public service offering routes, which are otherwise neglected without regulatory intervention. However, between two cities with the same ρ_i value, the city with a high valuation will win the slot. Moreover, our mechanism is not unfavorable to metro cities as our RCOF calculation; we also incorporated the population of the cities.

The metro cities with a higher population will have more slots. The policymaker can adjust the relative SPI weight and population to decide how much importance they want to give to remote connectivity. Moreover, when a city progresses on the economic front, its SPI value improves, leading to similar ρ_i values for most cities, reducing the difference given by $\gamma_{max} - \gamma_i$ ($SPI_{max} - SPI_i$), thus reducing the preferential treatment of the city. Overall, the social progress gap improves with the development of cities, and an administrative approach might not be required. Therefore, the ECATS will evolve and move toward a purely market-based mechanism.

Figure 7 and 8 show that increasing congestion costs leads to more average airline payments. Higher payments mean more internalization of airlines' congestion costs, which they earlier imposed on society, which has positive benefits to society regarding lower environmental and noise pollution and less waiting time for passengers. As shown, social utility decreases with an increase in congestion cost because we have only considered airline utilities in social utility calculations. Since airlines pay more by internalizing the congestion cost, their utilities go down. However, with reduced congestion, airlines will save more aircraft use, crew members and less fuel burn.

The social utility generated using ECATS is 20%-30% higher than PSAM and current allocation (Table 2). It is interesting to note that PSAM allocation performs better than the existing allocation by 10%-15%, whereas our mechanism outperforms PSAM. The properties of ECATS help us consider the congestion costs and RCOF in the objective function and incentivize a truthful value revelation by the airlines. The payment mechanism also considers congestion costs and RCOF, thereby driving airlines to operate in less congested slots and connect to remote cities. This study is the first to consider all three goals and provide a truthful and individually rational mechanism for airport slot allocation.

Table 2: Percent improvement in social utility of ECATS vis-a-vis Current and PSAM based allocations.

		Chennai (MAA) Airport		Delhi (DEL) Airport	
		% Improvement w.r.t.			
Time Interval	Congestion Cost (in INR)	Current allocation	PSAM allocation	Current allocation	PSAM allocation
6AM - 12PM	15000	29.9%	4.6%	35.1%	6.2%
	20000	28.4%	4.8%	33.5%	6.4%
	25000	26.6%	5.1%	31.7%	6.4%
	30000	24.3%	5.6%	30.4%	6.9%
12PM - 6PM	15000	29.9%	12.3%	43.2%	10.2%
	20000	29.1%	12.6%	42.4%	10.3%
	25000	28.4%	13.2%	41.1%	10.5%
	30000	27.0%	13.4%	39.6%	10.8%
6PM - 12AM	15000	36.9%	11.4%	34.2%	7.1%
	20000	37.8%	11.5%	33.3%	7.3%
	25000	38.9%	12.0%	31.9%	7.6%
	30000	39.9%	12.8%	29.9%	8.0%
12AM - 6AM	15000	28.7%	19.2%	32.1%	8.4%
	20000	29.7%	19.6%	30.5%	8.7%
	25000	31.0%	20.6%	28.5%	8.9%
	30000	32.4%	21.6%	26.5%	9.2%

7 Conclusions

This study examined the slot allocation process from multiobjective viewpoint. We started by considering of two major problems of the airline industry, congestion and regional connectivity. We combined the features of market-based and administrative instruments in a single dominant strategy incentive compatible mechanism with efficiency and social connectivity goals. The proposed mechanism provides a flexible and transparent approach to slot allocation and the analytical results offer meaningful policy implications. The mechanism is solvable in polynomial time. The study has several managerial and practical implications which are discussed.

First, the proposed mechanism provides a fine balance between three competing goals: efficiency, remote city connection and congestion mitigation. From an airport's perspective, the proposed model optimizes the slot allocation based on movement valuation and congestion costs. Therefore, airport managers have an efficient model that allocates slots to movements based on valuations. From a policymaker's perspective, the model also considers regional connectivity by incorporating RCOF. The population weights and SPI of the cities can be flexibly changed based on policymakers' goal. It provides equal opportunity to flights from metro and small cities because we find the average utility of flights comparable to the experimental results. Furthermore, the relationship between airport capacity and congestion is examined using variable capacity. It captured the phenomena of congestion when the capacity used exceeded its limits. The 'threshold limit' of airport capacity can be adjusted for different airports because it depends on factors such as aircraft types and technology. The 'threshold limit' could be an upper limit for the airport's uncongested capacity. Our mechanism brings in flexibility where:

- congestion and its associated cost can be tuned based on the airport's historical congestion data.
- threshold limit of capacity can be adjusted for different airports.
- the mechanism can be tweaked according to the objective of airport/priority of the policy maker, as it provides adjustable weights of SPI and population of the remote city, which caters to social obligations for slot allocation.

This study's second contribution comes from the methodology used in solving the proposed formulation. Solving the objective function for maximizing social welfare using the affine maximiser is an efficient and effective approach to mechanism design. The affine maximizer mechanisms are strategy-proof and individually rational (the agents' valuations for the chosen allocations are nonnegative). Therefore, our mechanism is incentive compatible with dominant strategies. No player can receive better utilities by misreporting their true valuation for the slots. The formulation is polynomial solvable because LP relaxation yields an optimal solution in integers in polynomial time.

This study's third contribution is its payment rule. The payment rule captures the externality imposed by a movement on the others. It is calculated as the difference between the social welfare generated without the movement and the sum of the weighted valuations of all other airlines if it participates in the allocation. The resulting payment captures two properties:

- The contribution of a specific movement to congestion
- The type of connectivity it provides (metro city, non-metro capital city or small city.)

Further, a policy maker viewpoint, the model aligns with the current practice of submitting slot requests at each airport. The slot requests currently submitted to airports contain information on flight numbers, arrival and departure times, and flight routes. The proposed model runs with only this information and does not require a change in this process. Regarding tricking the mechanism, we have proven that the only dominant strategy for airlines is to reveal their true values. The process relies on participants' truthfulness, which significantly contributes to policy making. The model also caters to the problem of airlines participation as we have proven the model's individual rationality property. Thus, `ECATS` is designed to solve airport's congestion and remote cities' connectivity problems simultaneously.

The mechanism is designed to assign low payments to remote city connections, encouraging airlines to operate on remote routes. Moreover, the high payment value for trunk routes discourages airlines from increasing frequency on only metro routes. The mechanism also considers the congestion externality imposed by a movement and charges a fee based on the movement's contribution. Hence, the payment mechanism emulate properties of the Pigouvian tax, used to correct inefficient markets by imposing a fee equal to the marginal cost of the negative externality. The proposed payment also drives our mechanism to be incentive-compatible and charge a higher cost to movements with negative externalities.

We have also shown that `ECATS` is weakly budget-balanced, meaning the planner does not require any external subsidy to operate and might even earn revenue. The sum of the payments made by all airlines will be either positive or equal to zero. In practice, airports earn revenue from aeronautical and commercial activities, where commercial operations contribute $65\% \pm 10\%$ percent of the total airport revenues. Sometimes, airports use concession revenue to subsidize aeronautical activities. Therefore, 'weakly budget-balance' is a sufficient and acceptable property for `ECATS` applications.

Limitations and future research directions

There are various possible ways to extend the work. Future studies can extend the literature on slot allocation by incorporating these limitations into their models. We considered a single airport for slot allocation and future studies might consider multiple airports for slot allocations simultaneously. Congestion in a single slot has a cascading effect on subsequent movements, and future studies might consider the rolling capacity constraint for different slots. In our remote city connectivity factor, we considered the SPI and population of the cities. Future studies could incorporate the availability of other transportation modes such as rail, road and water into these cities.

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Appendices

A Maintaining a minimum time gap between arrival and departure of the same movement (flight)

The arrival of a movement generally has an associated departure. The two movements are to be operated sequentially by the same aircraft, at the same airport, The time that passes from landing until take off for a new flight is called the turnaround time. During the turnaround time aircraft is serviced (i.e., cleaned, re-fuelled, etc.), and new passengers embark. We modify our model to include the turnaround time constraint for which both an arrival and a departure time have been stipulated. The arrival time is separated from a departure by at least a fixed time interval that we assume to be Δ.

To maintain the constraint of the turn around time between related arrival-departure flights, we define an $|M| \times |M|$ infeasibililty matrix denoted by $\mathcal{F}$. For $i, i' \in M$, if i is an arrival flight, i' is a departing flight and, if i' uses the same aircraft used by i then, $\mathcal{F}_{ii'} = 1$ otherwise $\mathcal{F}_{ii'} = 0$. If $\mathcal{F}_{ii'} = 1$, i' must be assigned to a slot only if i is assigned a slot and the slot assigned to i' must be after $j + \Delta$, where $\Delta \in \mathbb{Z}_{\geq 0}$. Notice that the matrix $\mathcal{F}$ can be decided before the mechanism as it is independent of the allocation and payment function.

A.1 The modified mechanism

The mechanism takes the arrival-departure constraint information of the movements as an input through the matrix $\mathcal{F}$.

Allocation function: The allocation function corresponding to the Equation (5) is as follows.

$$
\begin{aligned}
\underset{A}{\operatorname{argmax}} \quad & \sum_{j \in S} \sum_{i \in M} (\rho_i \; v_{ij} \; x_{ij}) \quad - \quad e_j(x) \; g \\
\text{subject to} \quad & \sum_{i \in M} x_{ij} \leqslant C_j \quad \forall j \in S, \\
& \sum_{j \in S} x_{ij} \leqslant 1 \quad \forall i \in M \\
& \mathcal{F}_{ii'} \left(x_{ij} + \sum_{j' \in S \cap [1, j+\Delta]} x_{i'j'} \right) \leqslant 1 \quad \forall i, i' \in M \quad \forall j \in S \\
& \mathcal{F}_{ii'} \left(\sum_{j' \in S} x_{i'j'} - \sum_{j \in S} x_{ij} \right) \leqslant 0 \quad \forall i, i' \in M \\
& x_{ij} = \{0, 1\} \quad \forall i \in M \quad \forall j \in S
\end{aligned}
\tag{15}
$$

In Equation (15), the first and second set of constraints are same as that in Equation (5). The third set of constraints ensures that a pair (i, i') of movements with $\mathcal{F}_{ii'} = 1$, the movements are never both allocated to the slots in range $(j, j + \Delta)$. In other words, if i is allocated to a slot j then, i' is never allocated a slot between 1 and $j + \Delta$. The fourth set of constraint ensures that for an arrival-departure pair of movements (i, i') where $\mathcal{F}_{ii'} = 1$, the departure flight i' is never assigned a slot unless the arrival flight i gets a slot.

The LP relaxation of Equation (15) after expanding the expression of e_j (as done in Equation (6)) is as follows:

$$
\begin{aligned}
&\underset{A,w}{\text{argmax}} \quad \sum_{j \in S} \sum_{i \in M} (\rho_i \; v_{ij} \; x_{ij}) \quad - \quad w_j \; g \\
&\text{subject to} \quad \sum_{i \in M} x_{ij} \leqslant C_j, \quad \forall j \in S \\
&\qquad \sum_{j \in S} x_{ij} \leqslant 1, \quad \forall i \in M \\
&\qquad \sum_{i \in M} x_{ij} - w_j \leqslant T_j, \quad \forall j \in S \\
&\mathcal{F}_{ii'} \left(x_{ij} + \sum_{j' \in S \cap [1, j+\Delta]} x_{i'j'} \right) \leqslant 1 \quad \forall i, i' \in M \quad \forall j \in S \\
&\mathcal{F}_{ii'} \left(\sum_{j' \in S} x_{i'j'} - \sum_{j \in S} x_{ij} \right) \leqslant 0 \quad \forall i, i' \in M \\
&\qquad x_{ij} \geqslant 0, \quad w_j \geqslant 0, \quad \forall i \in M, \;\; \forall j \in S
\end{aligned}
\tag{16}
$$

where, $T_j = C_j(1 - \lambda)$.

Payment function: The expression for payment function in the modified mechanism is same as that in Equation (7). However, to compute the allocation $A(\cdot)$ used in Equation (7), the Equation (16) is used.

A.2 Social and strategic guarantees

Notice that, the objective function of optimization problem (equation (16)) solved for allocation is same as that in Equation (6). This implies that our social goals such as, *egalitarianism, congestion-awareness* are maintained in the modified mechanism as the Equation (16) finds the optimal allocation among the feasible allocations that maximises the social utility (equation (4)).

As there is no change in the expression for payment and the objective function of the optimization problem for allocation, the proofs for Theorems 1 to 3 discussed in Section 5 remain intact. Hence, the theoretical guarantees regarding the implementation of the objectives, *truthfulness*, *individual rationality*, and *weak budget-balance* remain valid after the addition of the constraints to maintain the minimum time gap between arrival and departure of the same aircraft.

A.3 Experimental validation of total unimodularity

We have experimentally checked the total unimodularity of the coefficient matrix of Equation (16) on a variety of numeric input instances. We vary for the number of slots (n) from 1 to 500, number of movements (m) from 1 to 50, the minimum turn-around for movements using same aircraft (Δ) from 1 to n. For each instance of such (m, n, Δ), we arbitrary generate the matrix $\mathcal{F}$. The coefficient matrix has no dependency with the capacity of the slots hence, we fix the capacity of each slot to 10. Similarly, for other parameters (e.g., RCOF, per unit congestion cost, etc) which do not alter the coefficient matrix, we fix arbitrarily generated constant values to them. In the experiments we found that that, for all these input instances, the solution of Equation (16) is integral. This gives us some intuition that the coefficient matrix is TU, and if so, Equation (16) can be solved in polynomial time, and the solutions correspond to that of Equation (15). However, we leave the theoretical proof for the total unimodularity of the coefficient matrix of Equation (16) as a future exercise.