A Strict Ex-post Incentive Compatible Mechanism for Interdependent Valuations

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Abstract

The impossibility result by Jehiel and Moldovanu says that in a setting with interdependent valuations, any efficient and ex-post incentive compatible mechanism must be a constant mechanism. Mezzetti circumvents this problem by designing a two stage mechanism where the decision of allocation and payment are split over the two stages. This mechanism is elegant, however keeps a major weakness. In the second stage, agents are weakly indifferent about reporting their valuations truthfully: an agent's payment is independent of her reported valuation and truth-telling for this stage is by assumption. We propose a modified mechanism which makes truthful reporting in the second stage a strict equilibrium.

Keywords: Interdependent Value, Ex-post Incentive Compatibility
JEL: D51, D82

1. Introduction

In the classical independent private values model (Mas-Colell et al., 1995), each agent observes her valuation which depends on the allocation and her own private type. One can design efficient, dominant strategy incentive compatible mechanisms in this setting, e.g., the VCG mechanism achieves these properties. However, in many real world scenarios where agents collaboratively solve a problem, e.g., joint projects in multinational companies, or large distributed online projects, such as *crowdsourcing* experiments like the DARPA red balloon challenge (Pickard et al., 2011), the valuation of an agent depends on the private types of other agents as well. The model where the valuation of an agent for an allocation depends not only on her private type but also on the types of other agents, is the *interdependent value* model and has also received a great deal of attention in the literature (Krishna, 2009).

The interdependent value model poses a more difficult challenge for mechanism design. In increasing generality, Maskin (1992), Jehiel and Moldovanu (2001), and Jehiel et al. (2006) have shown that the efficient social choice function cannot *generically* be ex-post implemented. Ex-post implementation requires agents to be truthful about their own type reports when all others are reporting their types truthfully. This is a strong negative result - it rules out the existence of a mechanism that takes type reports from the agents and yields an allocation and a payment rule which satisfies ex-post incentive compatibility and efficiency. However, Mezzetti (2004) has shown that these goals can be achieved if the mechanism designer can split the allocation and payment decisions into two stages. The agents report their types in Stage 1, and the designer implements an allocation based on that. Then, each agent observes her own valuation and reports the values to the designer in Stage 2. The designer then proposes a payment based on the two-stage reports. This mechanism is called *generalized Groves mechanism* and in the Nash equilibrium the allocation is efficient. However, a drawback of the mechanism pointed out by Mezzetti (2004) is that the agents are indifferent between truth-telling and lying in Stage 2. Hereafter, we will refer to this mechanism as the *classic mechanism*.

In this paper, we propose a mechanism called Value Consistent Pivotal Mechanism (VCPM) that overcomes this difficulty. In particular it proposes a different set of payments from the classic mechanism in Stage 2 which makes it a strict ex-post Nash equilibrium for each agent to reveal her valuation truthfully at this stage.

The question may arise whether this mechanism hurts some other properties of the classic mechanism. Since VCPM yields the same payoff to the agents as that of the classic in equilibrium, it continues to satisfy all the properties that the classic mechanism satisfies in equilibrium. For example, we show that in a restricted problem domain, a refinement of our mechanism satisfies *individual rationality* (IR) as it is satisfied by the classic mechanism. However, truth-telling in the classic mechanism is also a subgame perfect equilibrium. On the contrary, truth-telling need not be a subgame perfect equilibrium in VCPM. We illustrate this with an example.

The rest of the paper is organized as follows. In Section 2, we introduce the model and define certain properties. In Section 3, we present the mechanism and discuss its properties. We conclude the paper in Section 4.

2. Model and Definitions

Let the set of agents be denoted by $N = \{1, \cdots, n\}$. Each agent observes her private type $\theta_i \in \Theta_i$. Let $\Theta = \times_{i \in N} \Theta_i$ denote the type profile space where $\theta \equiv (\theta_1, \cdots, \theta_n)$ be an element of Θ. We will denote the type profile of all agents except agent i by $\theta_{-i} \equiv (\theta_1, \cdots, \theta_{i-1}, \theta_{i+1}, \cdots, \theta_n) \in \Theta_{-i}$. Types are drawn independently across agents and each agent can only observe her own type and does not know the types of other agents. We consider a standard quasi-linear model. Therefore, the payoff u_i of agent i is the sum of her value v_i and transfer p_i.

We will consider a two stage mechanism similar to that of Mezzetti (2004), because of the impossibility result by Jehiel and Moldovanu (2001). We call this mechanism Value Consistent Pivotal Mechanism (VCPM).

In Stage 1, agents are asked to report their types, and after that the mechanism designer chooses an alternative from the set A via the allocation function $a : \Theta \to A$. We denote the reported types by $\hat{\theta}$, hence, the allocation for such a report is given by $a(\hat{\theta})$.

All agents then experience the consequence of the allocation via the valuation function which is defined by $v_i : A \times \Theta \to \mathbb{R}$, for all $i \in N$. Note: the value function is different from the independent private value setting, where it is a mapping $v_i : A \times \Theta_i \to \mathbb{R}$. This difference makes mechanism design in interdependent value settings difficult as discussed in Section 1.

In Stage 2, agents report their experienced valuations; transfers given by $p_i : \Theta \times \mathbb{R}^n \to \mathbb{R}$, $\forall i \in N$ are then decided by the designer. If the reported valuations are $\hat{v} \in \mathbb{R}^n$, the transfer to agent i is given by $p_i(\hat{\theta}, \hat{v})$.

The two stage mechanism VCPM is graphically illustrated in Figure 1.

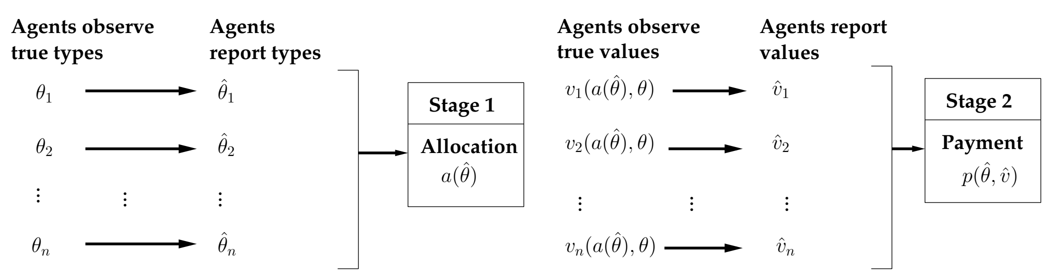

Figure 1: Graphical illustration of VCPM.

2.1. Definitions

As discussed earlier, in the paper we will consider only two stage mechanisms, where in Stage 1, agents report their types and in Stage 2, their experienced valuations. The allocation decision is made after Stage 1 and the payment after Stage 2. It is, therefore, necessary to define the notions of efficiency, truthfulness, and voluntary participation, in this setting.

We consider only quasi-linear domains where the payoff is the sum of the valuation and transfer. A mechanism M in this domain is fully characterized by a tuple of allocation and payment $\langle a, p \rangle$. For a truthful mechanism in this setting, we need to ensure that it is truthful in both stages. In Stage 1, truthfulness implies that the agents report their true types. In the second round, the valuation is a function of the allocation chosen in Stage 1. Here truthfulness would mean that they report their observed valuations due to that allocation.

The true type profile is given by θ. With a slight abuse of notation, we represent the true valuation vector by $v = (v_1(a(\theta), \theta), \cdots, v_n(a(\theta), \theta))$ under mechanism $M = \langle a, p \rangle$. Let us denote the payoff of agent i by $u_i^M(\hat{\theta}, \hat{v} | \theta, v)$ under mechanism M when the reported type and value vectors are $\hat{\theta}$ and $\hat{v}$ respectively, while the true type and value vectors are given by θ and v. Therefore, due to the quasi-linear assumption, the payoff is given by,

$$u_i^M(\hat{\theta}, \hat{v} | \theta, v) = v_i(a(\hat{\theta}), \theta) + p_i(\hat{\theta}, \hat{v}).$$

Definition 1 (Efficiency (EFF)). *A mechanism $M = \langle a, p \rangle$ is efficient if the allocation rule maximizes the sum valuation of the agents. That is, for all θ,*

$$a(\theta) \in \operatorname*{argmax}_{a \in A} \sum_{j \in N} v_j(a, \theta).$$

Definition 2 (Ex-post Incentive Compatibility (EPIC)). *A mechanism $M = \langle a, p \rangle$ is ex-post incentive compatible if reporting the true type and valuation is an ex-post Nash equilibrium of the induced game. That is, for all true type profiles $\theta = (\theta_i, \theta_{-i})$ and true valuation profiles $v = (v_i, v_{-i}) = (v_i(a(\theta), \theta), v_{-i}(a(\theta), \theta))$, and for all $i \in N$,*

$$\begin{aligned} & u_i^M((\theta_i, \theta_{-i}), (v_i(a(\theta), \theta), v_{-i}(a(\theta), \theta)) | \theta, v) \\ \geq \quad & u_i^M((\hat{\theta}_i, \theta_{-i}), (\hat{v}_i, v_{-i}(a(\hat{\theta}_i, \theta_{-i}), \theta)) | \theta, v), \ \forall \hat{\theta}_i, \hat{v}_i. \end{aligned}$$

Definition 3 (Ex-post Individual Rationality (EPIR)). *A mechanism $M = \langle a, p \rangle$ is ex-post individually rational if the payoff of each agent in the true type and valuation profile is non-negative. That is, for all $i \in N$, $\theta = (\theta_i, \theta_{-i})$, and $v = (v_i, v_{-i}) = (v_i(a(\theta), \theta), v_{-i}(a(\theta), \theta))$,*

$$u_i^M(\theta, v|\theta, v) \geq 0.$$

Subset Allocation (SA). Later in this paper, we will focus on a problem domain named *subset allocation*, where the allocation set is the set of all subsets of the agents, i.e., $A = 2^N$. In such a setting, we assume that the valuation of agent i is given by,

$$v_i(a, \theta) = \begin{cases} v_i(a, \theta_a) & \text{if } i \in a, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

We use θ_a to denote the type vector of the allocated agents, i.e., $\theta_a = (\theta_j)_{j \in a}, \forall a \in A$. This means that when agent i is not selected, her valuation is zero, and when she is selected, the valuation depends only on the types of the selected agents. This restricted domain is relevant for distributed projects in organizations, where the skill level of only the allocated agents matter in the value achieved by the other allocated agents. The skill levels of all the workers/employees participating in the project impact the success or failure of the project, and the reward or loss is shared by the participants of the project. This and several other examples of collaborative task execution falls under the SA domain, which makes it interesting to study.

3. Main Results

With the dynamics of the mechanisms as in Figure 1, the mechanism design problem is to design the allocation and the transfer rules. In VCPM, we adopt the following allocation and transfer rules.

Stage 1: Agents report types $\hat{\theta} = (\hat{\theta}_i, \hat{\theta}_{-i})$. The allocation is chosen as,

$$a^*(\hat{\theta}) \in \operatorname*{argmax}_{a \in A} \sum_{j \in N} v_j(a, \hat{\theta}). \tag{2}$$

Stage 2: Agents report valuations $\hat{v}$. The transfer to agent i, $i \in N$ is,

$$p_i^*(\hat{\theta}, \hat{v}) = \sum_{j \neq i} \hat{v}_j - g(\hat{v}_i, v_i(a^*(\hat{\theta}), \hat{\theta})) - h_i(\hat{\theta}_{-i}). \tag{3}$$

Where h_i is any arbitrary function of $\hat{\theta}_{-i}$, and $g(x, \ell)$ is a non-negative function of x with a unique zero at ℓ. An example of the function $g(x, \ell)$ would be $(x - \ell)^2$.

The stages of the mechanism are shown in algorithmic form in Algorithm 1. The difference between this mechanism with that of Mezzetti's is that we charge a tax to the agent i for not being consistent with Stage 1 of type reports. Note that the value function is common knowledge. Together with the reported type vector $\hat{\theta}$, the designer can compute the value $v_i(a^*(\hat{\theta}), \hat{\theta})$ in Eq. (3). The amount of tax is positive whenever the agents valuation announcement are inconsistent with the value computed according to their reported types in Stage 1. We will show in the following theorem that this modification in the transfer makes VCPM truth-telling a strict best-response in Stage 2.

Algorithm 1 VCPM

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Stage 1:
for agents i = 1, ..., n do
    agent i observes θ_i;
    agent i reports θ̂_i;
end for
compute allocation a*(θ̂) according to Eq. (2);
Stage 2:
for agents i = 1, ..., n do
    agent i observes v_i(a*(θ̂), θ);
    agent i reports v̂_i;
end for
compute payment to agent i, p*_i(θ̂, v̂), Eq. (3);
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Theorem 1. VCPM *is EFF and EPIC. In particular, reporting the valuations truthfully in Stage 2 of this mechanism is a strict best-response for each agent.*

Proof: The allocation rule of VCPM given by Eq. (2) ensures efficiency by construction. Therefore, we are only required to show that the mechanism is ex-post incentive compatible.

To show that VCPM is ex-post incentive compatible, without loss of generality, let us assume that all agents except agent i are reporting their types and valuations truthfully in the two stages. Let us assume that the true types are given by $\theta = (\theta_i, \theta_{-i})$. Hence, under these assumptions, $\hat{\theta} = (\hat{\theta}_i, \theta_{-i})$. The value reports in Stage 2 is dependent on the allocation in the first. Hence, for the agents $j \neq i$, who are truthful, the value reports are given by, $\hat{v}_j = v_j(a^*(\hat{\theta}), \theta)$. As defined earlier, we denote the payoff of agent

i by $u_i((\hat{\theta}_i, \theta_{-i}), (\hat{v}_i, v_{-i}(a(\hat{\theta}), \theta))|\theta, v)$ when all agents except i report their types and values truthfully, and the true type and value profiles are θ and v. The payoff of agent i is given by,

$$\begin{aligned}
&u_i^{\text{VCPM}}((\hat{\theta}_i, \theta_{-i}), (\hat{v}_i, v_{-i}(a^*(\hat{\theta}), \theta))|\theta, v) \\
&= v_i(a^*(\hat{\theta}_i, \theta_{-i}), \theta) + p_i((\hat{\theta}_i, \theta_{-i}), (\hat{v}_i, v_{-i})) \\
&= v_i(a^*(\hat{\theta}_i, \theta_{-i}), \theta) + \sum_{j \neq i} v_j(a^*(\hat{\theta}_i, \theta_{-i}), \theta) - g(\hat{v}_i, v_i(a^*(\hat{\theta}), \hat{\theta})) - h_i(\theta_{-i}) \\
&\leq v_i(a^*(\hat{\theta}_i, \theta_{-i}), \theta) + \sum_{j \neq i} v_j(a^*(\hat{\theta}_i, \theta_{-i}), \theta) - h_i(\theta_{-i}) \\
&= \sum_{j \in N} v_j(a^*(\hat{\theta}_i, \theta_{-i}), \theta) - h_i(\theta_{-i}) \\
&\leq \sum_{j \in N} v_j(a^*(\theta_i, \theta_{-i}), \theta) - h_i(\theta_{-i}) \\
&= v_i(a^*(\theta), \theta) + \sum_{j \neq i} v_j(a^*(\theta), \theta) - h_i(\theta_{-i}) \\
&= u_i^{\text{VCPM}}(\theta, v|\theta, v)
\end{aligned}$$

The first and second equalities are by definition and by substituting the expression of transfer (Eq. (3)). The first inequality comes since we are ignoring a non-positive term. The third equality is via simple reorganization the terms. The second inequality comes by definition of the allocation rule (Eq. (2)). The rest of the steps are simple reorganization of the expressions. The last equality is due to the fact that the function g is zero when $\hat{v}_i = v_i(a^*(\theta), \theta)$. Hence, we prove that VCPM is ex-post incentive compatible.

Let us explain that in this ex-post Nash equilibrium of this game, reporting values truthfully in Stage 2 is a strict best-response. This is because, when types are reported truthfully in Stage 1, the second term in the expression of the transfer for agent i (Eq. (3)) is reduced to $g(\hat{v}_i, v_i(a^*(\theta), \theta))$. This term is minimized (thereby the payoff to agent i is maximized) when $\hat{v}_i = v_i(a^*(\theta), \theta)$, which is the true report. Hence, truthful report in stage two of VCPM is a strict best-response. □

3.1. Comparison with the Classic Mechanism

It is reasonable to ask whether the proposed mechanism VCPM that achieves the strict Nash equilibrium in Stage 2, continues to satisfy all other desirable properties that the original mechanism given by Mezzetti (2004) used to satisfy. Since we prove that VCPM is EPIC, truthful reporting is an ex-post Nash equilibrium, and in that equilibrium, the penalty term $g(\hat{v}_i, v_i(a^*(\hat{\theta}), \hat{\theta}))$ in the expression of the transfer (Eq. (3)) is zero. Therefore, the allocation and the transfer in this truthful equilibrium are exactly the same as in the classic mechanism, and so is the payoff of each agent. So, any property that the classic mechanism used to satisfy in the ex-post Nash equilibrium will continue to hold even for VCPM. In addition, the truthful reporting in Stage 2 a strict best-response.

In the next section, we illustrate one such desirable property, namely the ex-post individual rationality, and show that under a restricted domain, a refined VCPM satisfies this property as does the classic mechanism.

It is important to note that the weak indifference in Stage 2 of the classic mechanism guarantees that truth reporting also a subgame perfect equilibrium. The strict EPIC of VCPM comes at the expense of subgame perfection. We illustrate this in the section following the next with an example.

3.2. VCPM *and Ex-post Individual Rationality*

In this section, we investigate the incentives for individuals to participate in this game. We consider the subset allocation (SA) domain. Hence, $A = 2^N$. Let us define the social welfare as,

$$W(\theta) = \max_{a \in A} \sum_{j \in a} v_j(a, \theta_a). \tag{4}$$

Similarly, the social welfare excluding agent i is given by,

$$W_{-i}(\theta_{-i}) = \max_{a_{-i} \in A_{-i}} \sum_{j \in a_{-i}} v_j(a_{-i}, \theta_{a_{-i}}), \tag{5}$$

where $A_{-i} = 2^{N \setminus \{i\}}$. Notice that in the SA domain $A_{-i} \subseteq A$, and therefore we make the following observation,

Observation 1. *In SA problem domain, with W and W_{-i} defined as in Eqs. (4) and (5), $W(\theta) \geq W_{-i}(\theta_{-i})$.*

This is because, $a_{-i} \in A_{-i} \subseteq A$. Therefore, while choosing allocation a that yields the social welfare including agent i, the designer has the choice of choosing all a_{-i}'s as well. Therefore, the social welfare including agent i will always dominate that excluding her.

Let us refine the VCPM to yield rVCPM by redefining the allocation (Eq. (2)) in stage one as follows.

$$a^{\text{rVCPM}}(\hat{\theta}) \in \underset{a \in A}{\operatorname{argmax}} \sum_{j \in a} v_j(a, \hat{\theta}_a). \tag{6}$$

We also redefine the payment (Eq. (3)) in stage two as follows.

$$p_i^{\text{rVCPM}}(\hat{\theta}, \hat{v}) = \sum_{j \in a^{\text{rVCPM}}(\hat{\theta}) \setminus \{i\}} \hat{v}_j - g(\hat{v}_i, v_i(a^{\text{rVCPM}}(\hat{\theta}), \hat{\theta})) - W_{-i}(\hat{\theta}_{-i}). \tag{7}$$

Note that the h_i function in the VCPM is replaced by W_{-i} in rVCPM. The following Corollary is now immediate from Theorem 1 and Observation 1.

Corollary 1. *In the SA problem domain,* rVCPM *is EFF, EPIC, and EPIR.*

Proof: Since rVCPM is a special case of VCPM and SA is a restricted domain, the results of VCPM holds in this setting too. Therefore from Theorem 1, we conclude that rVCPM is EFF and EPIC. Now, in the ex-post Nash equilibrium, the payoff of agent i is given by,

$$\begin{aligned}
&u_i^{\text{rVCPM}}(\theta, v|\theta, v) = v_i(a^{\text{rVCPM}}(\theta), \theta_{a^{\text{rVCPM}}(\theta)}) + p_i^{\text{rVCPM}}(\theta, v) \\
&= v_i(a^{\text{rVCPM}}(\theta), \theta_{a^{\text{rVCPM}}(\theta)}) \\
&\quad + \sum_{j \in a^{\text{rVCPM}}(\theta) \setminus \{i\}} v_j(a^{\text{rVCPM}}(\theta), \theta_{a^{\text{rVCPM}}(\theta)}) - W_{-i}(\theta_{-i}) \\
&= \sum_{j \in a^{\text{rVCPM}}(\theta)} v_j(a^{\text{rVCPM}}(\theta), \theta_{a^{\text{rVCPM}}(\theta)}) - W_{-i}(\theta_{-i}) \\
&= W(\theta) - W_{-i}(\theta_{-i}) \geq 0, \text{ (c.f. Observation 1).}
\end{aligned}$$

The first three equalities are by definition and simple reorganization of the terms. The fourth equality is by the definition of $W(\theta)$ (Eq. (4)) and $a^{\text{rVCPM}}(\theta)$ (Eq. (7)). The inequality is due to Observation 1. Hence, rVCPM is ex-post individually rational. □

3.3. VCPM *and Subgame Perfect Equilibrium*

In this section, we show with an example that truth reporting in VCPM is not a subgame perfect equilibrium (SPE). [1]

[1] We thank an anonymous referee for raising this point and providing the example.

Example 1. *Let us consider two alternatives* a_1 *and* a_2 *and three agents,* $i = 1, 2, 3$*. Let* $\Theta_1 = \Theta_2 = \{0, 2\}$*, and* $\Theta_3 = \phi$ *(i.e., agents* 1 *and* 2 *have two types each, agent* 3 *has no private information). Assume,* $v_i(a_1, \theta) = 0$*, for all* i *and* θ*,* $v_1(a_2, \theta) = \theta_1 + 2\theta_2$*,* $v_2(a_2, \theta) = 2\theta_1 + \theta_2$*,* $v_3(a_2, \theta) = -3$*, for all* θ*. The efficient allocation under* VCPM *is,* $a^*(\theta) = a_1$*, if* $\theta_1 = \theta_2 = 0$*, and* a_2 *otherwise. Let us assume,* $h_i \equiv 0$ *for simplicity. Under* VCPM*, no matter what was reported in Stage 1, the unique best reply for each agent is to report a value,* $\hat{v}_i = v_i(a^*(\hat{\theta}), \hat{\theta})$*. Now suppose that the true types are* $\theta_1 = \theta_2 = 0$ *and that agent* 2 *truthfully reports in Stage 1, i.e.,* $\hat{\theta}_2 = 0$ *and then reports* $\hat{v}_2 = v_2(a^*(\hat{\theta}), \hat{\theta})$ *(as it must in a SPE).*

If agent 1 *reports the truth, i.e.,* $\hat{\theta}_1 = 0$*, then the implemented allocation is* $a^*(0, 0) = a_1$*. The allocation gives him zero value, and the second round transfer to agent* 1 *is* $\sum_{j \neq 1} v_j(a_1, (0, 0)) = 0$*. In short, agent* 1 *obtains a total utility of zero if he reports his true type.*

If he instead lies and reports $\hat{\theta}_1 = 2$*, then the implemented allocation is* $a^*(2, 0) = a_2$*. The allocation still gives him zero value (as the true types are* $\theta_1 = \theta_2 = 0$*), but the second round transfer to agent* 1 *now is* $\sum_{j \neq 1} v_j(a_2, (2, 0)) = 4 - 3 = 1$*. In short, agent* 1 *obtains a total utility of one if he misreports. So, truth-telling in Stage 1 is not a SPE for agent* 1*.*

The classic mechanism is SPE because the payoffs of the agents in Stage 2 are independent of their value reports, thereby making the EPIC weak. Hence, we can see that there is a trade-off between strict EPIC in Stage 2 and subgame perfection.

3.4. Reduced Form vs State-of-the-World Formulation

In the original paper by Mezzetti (2004), a *state-of-the-world* variable ω was introduced, which is realized after the allocation and before the agents observe their valuations. The valuations are functions of this variable, and the payment is decided after they report their observed valuations that depend on the realization of ω. The mechanism proposed in that paper is weak EPIC in Stage 2 since the payment does not depend on the agent concerned's Stage 1 report. The EPIC in the first round, however, is with the expectation over the ω, since the allocation decision is done in Stage 1 and before ω realizes. In contrast, the *reduced form* refers to the setting where all the analysis is done taking expectation over ω. In this paper, we have discussed the reduced form analysis so far, and now we make a few observations on the state-of-the-world formulation.

The state-of-the-world variable ω affects any strict EPIC mechanism including VCPM in the following way. It depends on the following cases regarding what the designer observes.

Case 1: *the designer can observe the state-of-the-world* ω. In such a case, by redefining the penalty term to be $g(\hat{v}_i, v_i(a^*(\hat{\theta}), \hat{\theta}, \omega))$, where $v_i(a^*(\hat{\theta}), \hat{\theta}, \omega)$ is now computable by the designer, we can satisfy the strict EPIC in the second round for each realization of ω, in addition to satisfying EPIC in the first round with expectation over ω.

Case 2: *the designer cannot observe the state-of-the-world* ω*, but is able to decipher it given the agents reports.* This is going to hold for a restricted class of problems. For example, if for each allocation a, type profile θ, and $x \in \text{Range}(v)$, there exists at least two agents $i_1(x)$ and $i_2(x)$ such that both $v_{i_1(x)}^{-1}(x)$ and $v_{i_2(x)}^{-1}(x)$ gives back a, θ, and a unique ω. This implies that, each state of the world uniquely affects at least two agents in the population. In such a setting, it is possible for the designer to retrieve the true state-of-the-world ω from the other agents' report and use the g function as used in Case 1 above, and make VCPM a strict EPIC in Stage 2 as well.

Case 3: *the designer cannot observe the state-of-the-world* ω*, and is not able to decipher it given the agents reports.* This scenario is difficult for designing any strict EPIC mechanism. We provide an example where a naive expectation of g given by, $\int_\Omega g(\hat{v}_i, v_i(a^*(\hat{\theta}), \hat{\theta}, \omega))\, \mathrm{d}\omega$ does not work, and leave a detailed investigation to future work.

Example 2. *Suppose,* ω *can take only two possible states,* 1 *and* 0*. The priors are* $\mathbb{P}(\omega = 1) = 0.99$*, and* $\mathbb{P}(\omega = 0) = 0.01$*. Let us fix an allocation* a *and a type profile* θ*, and let* $v_i(a, \theta, \omega = 1) = 1$*, and* $v_i(a, \theta, \omega = 0) = 0$*, then if* i *observes* $v_i = 0$*, she might still report* $v_i = 1$ *because the* ω *corresponding to that outcome is a lot more likely. Say* $g(x, \ell) = (x - \ell)^2$*, the agent would look at minimizing* $(\hat{v}_i - 1)^2 \times 0.99 + (\hat{v}_i - 0)^2 \times 0.01$ *which is minimized by* $\hat{v}_i = 1$ *even if* $v_i = 0$*.*

The example above clearly shows that there is no easy way to convert a strict EPIC mechanism in the reduced form into a strict EPIC mechanism with state-of-the-world formulation. It appears to us that in an unrestricted interdependent value domain, it may be impossible to design any strict EPIC mechanism, an investigation which we leave to future work.

4. Conclusions and Future Work

In this paper, we have considered mechanisms for interdependent valuation settings. The classic mechanism by Mezzetti is ex-post incentive compatible for such settings, but in the second stage of valuation reporting is truthful in the weakest possible sense, as payments to agents are independent of their own reported valuations. We improve the classic mechanism by making truthful reporting in the second stage a strict Nash equilibrium, though it comes at the cost of subgame perfection of the classic mechanism. It will be interesting to investigate if these two properties are possible to satisfy together. Our analysis is based on the reduced form formulation of the classic mechanism. There is another interesting strand of work, which would include the state-of-the-world formulation as well and find the feasible space of mechanism design. We conclude our paper keeping these as our potential future works.

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